H29 PARABOLEN HAVO

29.0 INTRO

1 a b

29.1 PARABOLEN

2 a $6 - x$
  b $y = x(6 - x)$
  c

x	-1	0	1	2	3	4	5	6	7
y	-7	0	5	8	9	8	5	0	-7

  d …
  e $x = 3$

3 a $y = x(x + 5)$
  b

x	-6	-5	-4	-3	-2	-1	0	1	2
y	6	0	-4	-6	-6	-4	0	6	14
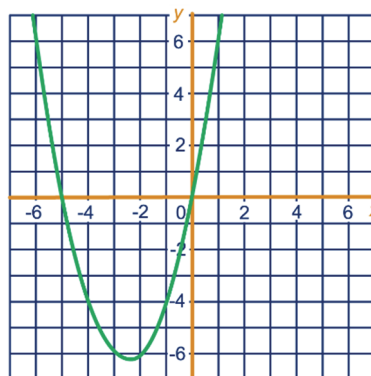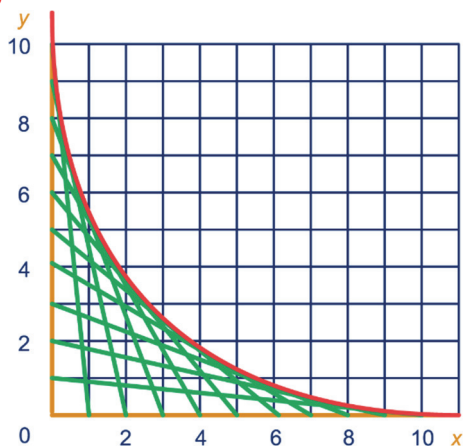
c …
d $x = -2\frac{1}{2}$

4 $(3, 9)$; $(-2\frac{1}{2}, -6)$

5 a

x	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = \frac{1}{10}x^2$	0,9	0,4	0,1	0	0,1	0,4	0,9
$y = \frac{1}{2}x^2$	4,5	2	0,5	0	0,5	2	4,5
$y = 2x^2$	18	8	2	0	2	8	18

b d
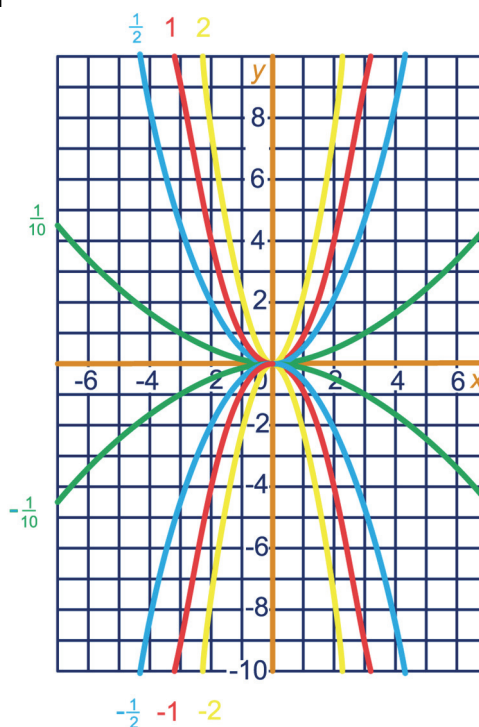
c

x	-3	-2	-1	0	1	2	3
$y = -x^2$	-9	-4	-1	0	-1	-4	-9
$y = -\frac{1}{10}x^2$	-0,9	-0,4	-0,1	0	-0,1	-0,4	-0,9
$y = -\frac{1}{2}x^2$	-4,5	-2	-0,5	0	-0,5	-2	-4,5
$y = -2x^2$	-18	-8	-2	0	-2	-8	-18

e Dalparabool als $c > 0$,
  een bergparabool als $c < 0$.
f Ze zijn elkaars spiegelbeeld in de x-as.
g Dan is $y = 0$, dat is een rechte lijn, dat is de
  vergelijking van de x-as.

6 $y = cx^2$
$3 = c \cdot 1^2$ (invullen het punt (1, 3))
$3 = c$

$y = cx^2$
$2 = c \cdot (-5)^2$ (invullen het punt (-5, 2))
$2 = 25c$
$\frac{2}{25} = c$

$y = cx^2$
$-3 = c \cdot 3^2$ (invullen het punt (3, -3))
$-3 = 9c$
$-\frac{1}{3} = c$

7 $y = cx^2$
$4 = c \cdot 5^2$ (invullen het punt (5, 4) of (-5, 4))
$4 = 25c$
$\frac{4}{25} = c$

29.2 HERHALING

8 $12 - 18x$

$8x^2 - 20x$

$-4x + 10x^2$

$3x - 8$

$-2x^2 - 39x - 10$

$15x^2 - 9x - 13$

$22 + 2x^2 - 6x$

$-3x + 4y$

$5x + 3y$

9 $x^2 - 10x + 21$ $3x^2 + 5x - 8$

$6x^2 + 22x - 8$ $2x^2 + 3x - 2$

$p^2 + 6p + 9$ $25 - 10q + q^2$

$-p^2 + 4q^2$ $4p^2 - 12pq + 9q^2$

10 $x(x + 7)$ $(x - 3)^2$

$x(x - 10)$ $(x + 5)^2$

$(x - 7)(x - 1)$ $(x - 6)^2$

$(x - 3)(x + 9)$ $(2x - 3)^2$

11 $x^2 + 10x = -16$
$x^2 + 10x + 16 = 0$ ➤ PLUS 16
$(x + 2)(x + 8) = 0$ ➤ ONTBINDEN
$x = -2$ of $x = -8$

$10x = x^2$
$10x - x^2 = 0$ ➤ MIN x^2
$x(10 - x) = 0$ ➤ ONTBINDEN
$x = 0$ of $x = 10$

$x^2 + 6x = 16$ ➤ MIN 16
$x^2 + 6x - 16 = 0$ ➤ ONTBINDEN
$(x + 8)(x - 2) = 0$
$x = -8$ of $x = 2$

$x^2 + 16 = 8x$ ➤ MIN $8x$
$x^2 - 8x + 16 = 0$ ➤ ONTBINDEN
$(x - 4)^2 = 0$
$x = 4$

$3(x + 1) = x^2 + 5$ ➤ HAAKJES WEG
$3x + 3 = x^2 + 5$ ➤ MIN $3x$, MIN 3
$x^2 - 3x + 2 = 0$ ➤ ONTBINDEN
$(x - 2)(x - 1) = 0$
$x = 2$ of $x = 1$

$(x + 1)(x + 3) = 1 - x^2$ ➤ HAAKJES WEG
$x^2 + 4x + 3 = 1 - x^2$ ➤ PLUS x^2, MIN 1
$2x^2 + 4x + 2 = 0$ ➤ DELEN DOOR 2
$x^2 + 2x + 1 = 0$ ➤ ONTBINDEN
$(x + 1)(x + 1) = 0$
$x = -1$

$x^2 - 5x = 6$ ➤ MIN 6
$x^2 - 5x - 6 = 0$ ➤ ONTBINDEN
$(x - 6)(x + 1) = 0$
$x = 6$ of $x = -1$

$3 - 4x = 1 - 2x^2$ ➤ PLUS $2x^2$, MIN 1
$2x^2 - 4x + 2 = 0$ ➤ DELEN DOOR 2
$x^2 - 2x + 1 = 0$ ➤ ONTBINDEN
$(x - 1)(x - 1) = 0$
$x = 1$

$12 - 11x = x^2$ ➤ PLUS $11x$, MIN 12
$x^2 + 11x - 12 = 0$ ➤ ONTBINDEN
$(x + 12)(x - 1) = 0$
$x = -12$ of $x = 1$

$3x^2 = 6x - 3$ ➤ MIN $6x$, PLUS 3
$3x^2 - 6x + 3 = 0$ ➤ DELEN DOOR 3
$x^2 - 2x + 1 = 0$ ➤ ONTBINDEN
$(x - 1)(x - 1) = 0$
$x = 1$

$5x^2 = -15x$ ➤ PLUS $15x$
$5x^2 + 15x = 0$ ➤ DELEN DOOR 5
$x^2 - 3x = 0$ ➤ ONTBINDEN
$x(x - 3) = 0$
$x = 0$ of $x = 3$

$2(x^2 - 2) = 4(x^2 - 3)$ ➤ HAAKJES WEG
$2x^2 - 4 = 4x^2 - 12$ ➤ MIN $2x^2$, PLUS 4
$0 = 2x^2 - 8$ ➤ DELEN DOOR 2
$0 = x^2 - 4$ ➤ ONTBINDEN
$0 = (x - 2)(x + 2)$
$x = 2$ of $x = -2$

29.3 PARABOLEN TEKENEN

12 a $h = 0{,}0625 \cdot 40^2 = 100$ m

   b als $x = 0$, $h = 0{,}0625 \cdot 0^2 = 0$ m
       als $x = 10$, $h = 0{,}0625 \cdot 10^2 = 6{,}25$ m
       als $x = 20$, $h = 0{,}0625 \cdot 20^2 = 25$ m
       als $x = 30$, $h = 0{,}0625 \cdot 30^2 = 56{,}25$ m
       als $x = 40$, $h = 0{,}0625 \cdot 40^2 = 100$ m

   c $x = 35$, dan $h = 0{,}0625 \cdot 35^2 = 76{,}5625$ m
       De hoogte boven de Wupper is dan
       $100 - 76{,}5625 = 23{,}4375$ m.

13 a $x^2 - 6x = 0$
       $x(x - 6) = 0$
       $x = 0$ of $x = 6$
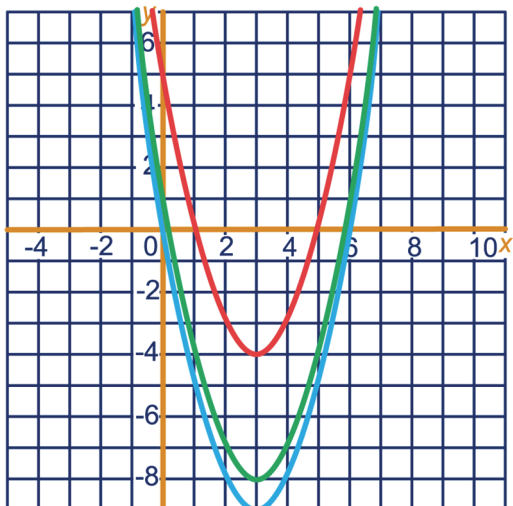   Dus de nulpunten zijn 0 en 6.

   b vergelijking symmetrieas: $x = \frac{0+6}{2} = 3$

   c $y = 3^2 - 6 \cdot 3 = -9$, Top(3, -9).

   d

x	-1	0	$\frac{1}{2}$	1	2	3	4	5	$5\frac{1}{2}$	6	7
y	7	0	$-2\frac{3}{4}$	-5	-8	-9	-8	-5	$-2\frac{3}{4}$	0	7

   ef

   g 9
   h $y = x^2 - 6x + 9 = (x - 3)^2$

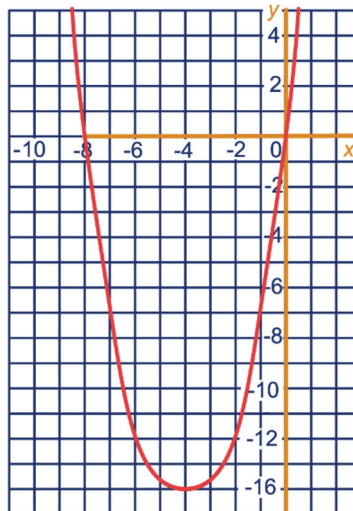**14 a** $x^2 + 8x = 0$
       $x(x + 8) = 0$
       $x = 0$ of $x = -8$
       Dus de nulpunten zijn 0 en -8.

   b vergelijking symmetrieas: $x = \frac{-8+0}{2} = -4$

   c $y = (-4)^2 + 8 \cdot -4 = -16$
       Top(-4, -16).

   d

x	-9	-8	-7	-6	-5	-4	-3	-2	-1	0	1
y	9	0	-7	-12	-15	-16	-15	-12	-7	0	9

   e 16
   f $y = x^2 + 8x + 16 = (x + 4)^2$

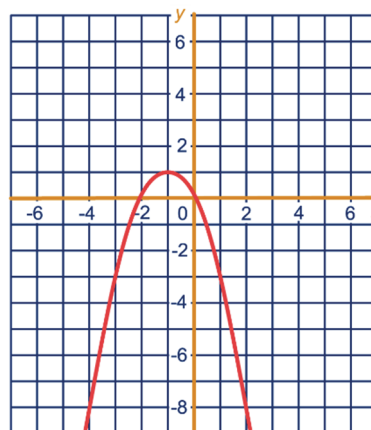**15 a** $-x^2 - 2x = 0$
       $-x(x + 2) = 0$
       $x = 0$ of $x = -2$
       Dus de nulpunten zijn 0 en -2.

   b vergelijking symmetrieas: $x = \frac{-2+0}{2} = -1$

   c $y = -(-1)^2 - 2 \cdot -1 = -1 + 2 = 1$
       Top(-1, 1).

   d

x	-6	-5	-4	-3	-2	-1	0	1	2	3	4
y	-24	-15	-8	-3	0	1	0	-3	-8	-15	-24

   e -1

16 a snijpunt y-as $\Rightarrow x = 0$
       $y = 0^2 - 2 \cdot 0 + 4 = 4$
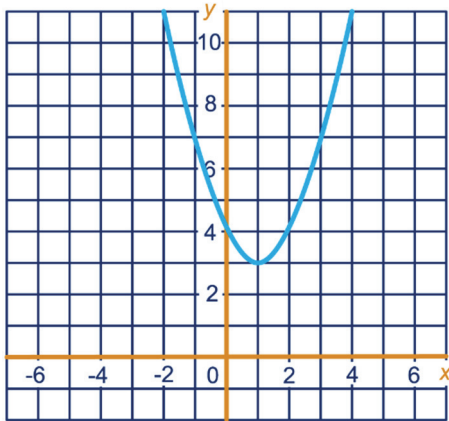   (0,4)

   b $x^2 - 2x + 4 = 4$
       $x^2 - 2x = 0$
       $x(x - 2) = 0$
       $x = 0$ of $x = 2$

   c vergelijking symmetrieas: $x = \frac{0+2}{2} = 1$

   d $y = 1^2 - 2 \cdot 1 + 4 = 3$
       Top(1, 3).

e

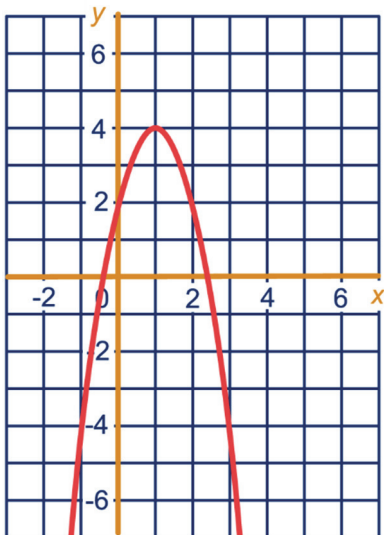| x	-2	-1	0	1	2	3	4
y	12	7	4	3	4	7	12

17 a $y = -2 \cdot 0^2 + 4 \cdot 0 + 2 = 2$
Snijpunt y-as (0, 2).

b $-2x^2 + 4x + 2 = 2$

$-2x^2 + 4x = 0$

$x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$ of $x = 2$

c vergelijking symmetrieas: $x = \frac{0+2}{2} = 1$

d $y = -2 \cdot 1^2 + 4 \cdot 1 + 2 = 4$
Top(1, 4).

e

18 $y = x^2 - 2x$
Nulpunten:

$x^2 - 2x = 0$

$x(x - 2) = 0$

$x = 0$ of $x = 2$

Snijpunt $y-$as:

$y = 0^2 + 2 \cdot 0 = 0$

(0,0)

Symmetrieas:

$x = \frac{0+2}{2} = 1$

$y = 1^2 - 2 \cdot 1 = -1$

Top(1, -1).

x	-2	-1	0	1	2	3	4
y	8	3	0	-1	0	3	8

$y = -x^2 + 5x - 2$
Snijpunt $y-$as:

$y = -0^2 + 5 \cdot 0 - 2 = -2$

(0, -2)

$-x^2 + 5x - 2 = -2$

$-x^2 + 5x = 0$

$-x(x - 5) = 0$

$x = 0$ of $x = 5$

Symmetrieas:

$x = \frac{0+5}{2} = 2\frac{1}{2}$

$y = -(2\frac{1}{2})^2 + 5 \cdot 2\frac{1}{2} - 2 = 4\frac{1}{4}$

Top($2\frac{1}{2}$, $4\frac{1}{4}$).

x	-1	0	1	2	$2\frac{1}{2}$	3	4	5	6
y	-8	-2	2	4	$4\frac{1}{4}$	4	2	-2	-8

$y = (x + 2)(x - 8)$
Nulpunten:

$(x + 2)(x - 8) = 0$

$x = -2$ of $x = 8$

Snijpunt $y-$as:

$y = (0 + 2)(0 - 8) = -16$

(0, -16)

Symmetrieas:

$x = \frac{-2+8}{2} = 3$

$y = (3 + 2)(3 - 8) = -25$

Top(3, -25).

x	0	1	2	3	4	5	6
y	-16	-21	-24	-25	-24	-21	-16

$y = 2x^2 - 3x + 1$

Snijpunt y – as:

$y = 2 \cdot 0^2 - 3 \cdot 0 + 1 = 1$

$(0, 1)$

$2x^2 - 3x + 1 = 1$

$2x^2 - 3x = 0$

$2x(x - 1\frac{1}{2}) = 0$

$x = 0$ of $x = 1\frac{1}{2}$

Symmetrieas:

$x = \frac{0 + 1\frac{1}{2}}{2} = \frac{3}{4}$

$y = 2 \cdot (\frac{3}{4})^2 - 3 \cdot \frac{3}{4} + 1 = -\frac{1}{8}$

Top$(\frac{3}{4}, -\frac{1}{8})$.

x	-2	-1	0	$\frac{3}{4}$	1	2	3
y	15	6	1	$-\frac{1}{8}$	0	3	10

$y = -3x^2 - 9x + 6$

Snijpunt y – as:

$y = -3 \cdot 0^2 - 9 \cdot 0 + 6 = 6$

$(0, 6)$

$-3x^2 - 9x + 6 = 6$

$-3x^2 - 9x = 0$

$x^2 + 3x = 0$

$x(x + 3) = 0$

$x = 0$ of $x = -3$

Symmetrieas:

$x = \frac{0 + -3}{2} = -1\frac{1}{2}$

$y = -3 \cdot (-1\frac{1}{2})^2 - 9 \cdot -1\frac{1}{2} + 6 = 12\frac{3}{4}$

Top$(-1\frac{1}{2}, 12\frac{3}{4})$.

x	-4	-3	-2	$-1\frac{1}{2}$	-1	0	1
y	-6	6	12	$12\frac{3}{4}$	12	6	-6

$y = x(x - 8) + 2$

Snijpunt y – as :

$y = 0 \cdot (0 - 8) + 2 = 2$

$(0,2)$
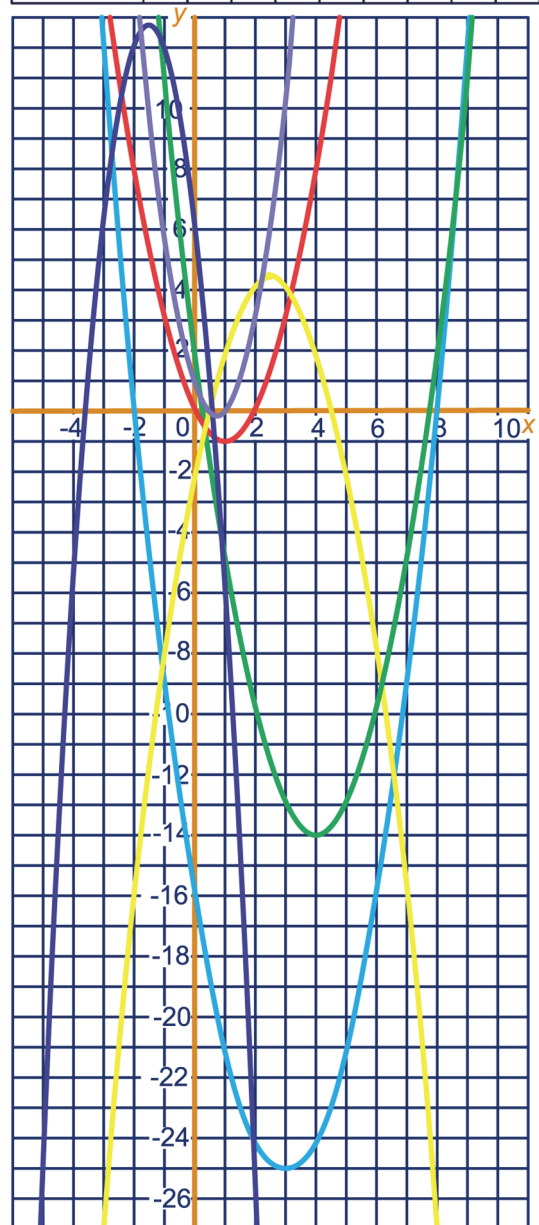
$x(x - 8) + 2 = 2$

$x(x - 8) = 0$

$x = 0$ of $x = 8$

Symmetrieas:

$x = \frac{0 + 8}{2} = 4$

$y = 4 \cdot (4 - 8) + 2 = -14$

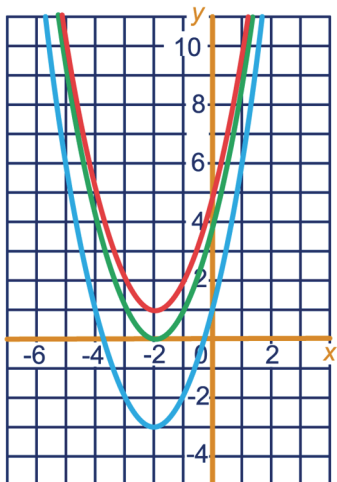Top$(4, -14)$.

x	-1	0	1	2	3	4	5	6	7
y	11	2	-5	-10	-13	-14	-13	-10	-5

19 a $(0, 0)$

  b $(0, 1)$; $(0, -3)$

  c $y \geq 0$

  d $(-2, 0)$

  e $(-2, 1)$; $(-2, -3)$

  f

x	-5	-4	-3	-2	-1	0	1
$y = (x + 2)^2$	9	4	1	0	1	4	9
$y = (x + 2)^2 + 1$	10	5	2	1	2	5	10
$y = (x + 2)^2 - 3$	6	1	-2	-3	-2	1	6

20 a Omdat $2(x-1)^2$ voor elke waarde van x, 0 of meer is. Dus $2(x-1)^2 + 3$ is minimaal 3.

  b $y \geq 3$

  c Een dalparabool, omdat $2(x-1)^2$ minimaal 0 is.

  d bijv. $y = 2(x-1)^2$

  e Door bijv. van $2(x-1)^2$ een getal af te trekken.

  f vergelijking symmetrieas: $x = 1$

  g Top(1, 3).

  h
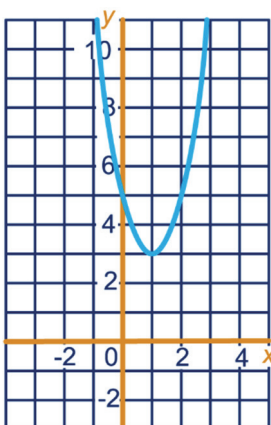
21 a Een bergparabool, omdat $-2(x+1)^2$ maximaal 0 is,

  b Twee nulpunten, omdat de top op hoogte 6 ligt en het een bergparabool is.

  c vergelijking symmetrieas: $x = -1$

  d Top(-1, 6).

  e
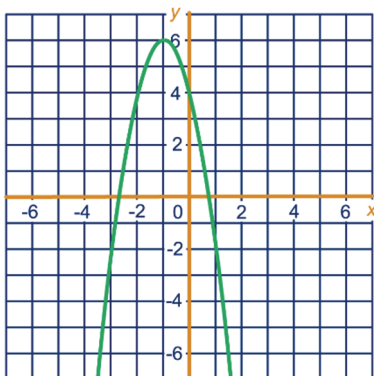
22 a Dalparabool als $c > 0$, een bergparabool als $c < 0$.

  b (a, b)

23 (-2, -3) en (9, 19)

29.4 TOEPASSINGEN

24 a $x - \frac{1}{100}x^2 = 0$

  $100x - x^2 = 0$

  $x(100 - x) = 0$

  $x = 0$ of $x = 100$

  Dus de kogel legt 100 meter af.

  b Vanwege symmetrie wordt de grootste hoogte bereikt als $x = 50$.

  Dan $y = 50 - \frac{1}{100} \cdot 50^2 = 25$, dus 25 meter.

25 a $C = 3 \cdot 1\frac{1}{2} \cdot 10 = 45$

  b $C = x(6 - 2x) \cdot 10 = 60x - 20x^2$

  c een bergparabool

  d Nulpunten:

  $60x - 20x^2 = 0$

  $20x(3 - x) = 0$

  $x = 0$ of $x = 3$

  Top ligt bij $x = 1\frac{1}{2} \Rightarrow y = 60 \cdot 1\frac{1}{2} - 2 \cdot (1\frac{1}{2})^2 = 45$

  Top($1\frac{1}{2}$, 45).

  e Bij hoogte $1\frac{1}{2}$ dm, de capaciteit is dan 45 liter.

26 $y = cx^2$

  $62,5 = c \cdot 250^2$ (invullen het punt (250; 62,5))

  $62,5 = 62.500c$

  $\frac{1}{1000} = c$

  Vergelijking parabool: $y = \frac{1}{1000}x^2$

27 a $(10 + 2)^2 - 10 - 10 = 124$ stippen

  b $(n + 2)^2 - 2n = n^2 + 2n + 4$

  c $n^2 + 2n + 4 = 10204$

  $n^2 + 2n - 10200 = 0$

  $(n - 100)(n + 102) = 0$

  $n = 100$ of $n = -102$

  Alleen $n = 100$ voldoet, omdat $n > 0$ moet zijn.

  d $n^2 + 2n + 4 + 43 = (n + 1)^2 + 2(n + 1) + 4$

  $n^2 + 2n + 47 = n^2 + 4n + 7$

  $40 = 2n$

  $20 = n$

  Rangnummer is 20. Dus rangnummer 21 heeft 43 stippen meer dan rangnummer 20.

28 a $(60 - 2x)^2 = 1000$

$60 - 2x = \sqrt{1000}$ of $60 - 2x = -\sqrt{1000}$

$2x = 60 - \sqrt{1000}$ of $2x = 60 + \sqrt{1000}$

$2x = 60 - 10\sqrt{10}$ of $2x = 60 + 10\sqrt{10}$

$x = 30 - 5\sqrt{10}$ of $x = 30 + 5\sqrt{10}$

$x \approx 14,19$ cm of $x \approx 45,81$ cm

Alleen $x = 30 - 5\sqrt{10} \approx 14,19$ cm voldoet, omdat $2x < 60$, dus $x < 30$ moet zijn.

b $(60 - 2x)^2 = 4 \cdot x \cdot (60 - 2x)$

$3600 - 240x + 4x^2 = 240x - 8x^2$

$12x^2 - 480x + 3600 = 0$

$x^2 - 40x + 300 = 0$

$(x - 30)(x - 10) = 0$

$x = 30$ of $x = 10$

Alleen $x = 10$ cm voldoet, omdat $x < 30$ moet zijn.

29 a $-t^2 + 4t + 21 = 21$

$-t^2 + 4t = 0$

$t^2 - 4t = 0$

$t(t - 4) = 0$

$t = 0$ of $t = 4$

Dus na 4 sec. is het steentje weer op dezelfde hoogte.

b $-t^2 + 4t + 21 = 0$

$t^2 - 4t - 21 = 0$

$(t - 7)(t + 3) = 0$

$t = 7$ of $t = -3$

Dus na 7 sec. bereikt het steentje het water.

c vergelijking symmetrieas: $t = \frac{7-3}{2} = 2$

$h = -2^2 + 4 \cdot 2 + 21 = 25$

Op 2 sec. bereikt het steentje de maximale hoogte van 25 m.

30 a $2l + 2b = 44 \Rightarrow l + b = 22$

b $l + b = 22 \Rightarrow l = 22 - b$

$(22 - b) \cdot b = 120$

$22b - b^2 = 120$

$b^2 - 22b + 120 = 0$

$(b - 12)(b - 10) = 0$

$b = 12$ of $b = 10$

Als $b = 12$, dan $l = 22 - 12 = 10$.

Als $b = 10$, dan $l = 22 - 10 = 12$.

De rechthoek is 12 bij 10 of de rechthoek is 10 bij 12.

c Stelsel:
$$\begin{cases} l \cdot b = 720 \\ l = 5b \end{cases}$$

Vergelijking:

$5b \cdot b = 720$

$5b^2 = 720$

$b^2 = 144$

$b = 12$ of $b = -12$

Alleen $b = 12$, voldoet omdat $b > 0$ moet zijn.

Dan $l = 5 \cdot 12 = 60$.

Dus de breedte is 12 en de lengte 60.

29.5 VERGELIJKINGEN OPLOSSEN

31 $(x - 3)^2 = 100$

$x - 3 = 10$ of $x - 3 = -10$

$x = 13$ of $x = -7$

$(x + \frac{1}{2})^2 = 25$

$x + \frac{1}{2} = 5$ of $x + \frac{1}{2} = -5$

$x = 4\frac{1}{2}$ of $x = -5\frac{1}{2}$

$(2x + 1)^2 = 36$

$2x + 1 = 6$ of $2x + 1 = -6$

$2x = 5$ of $2x = -7$

$x = 2\frac{1}{2}$ of $x = -3\frac{1}{2}$

$(x - 3)^2 = 13$

$x - 3 = \sqrt{13}$ of $x - 3 = -\sqrt{13}$

$x = 3 + \sqrt{13}$ of $x = 3 - \sqrt{13}$

$(x + \frac{1}{2})^2 = 48$

$x + \frac{1}{2} = \sqrt{48} = 4\sqrt{3}$ of $x + \frac{1}{2} = -\sqrt{48} = -4\sqrt{3}$

$x = -\frac{1}{2} + 4\sqrt{3}$ of $x = -\frac{1}{2} - 4\sqrt{3}$

$(2x + 1)^2 = 68$

$2x + 1 = \sqrt{68} = 2\sqrt{17}$ of $2x + 1 = -\sqrt{68} = -2\sqrt{17}$

$2x = -1 + 2\sqrt{17}$ of $2x = -1 - 2\sqrt{17}$

$x = -\frac{1}{2} + \sqrt{17}$ of $x = -\frac{1}{2} - \sqrt{17}$

32 25 ; 5   7 ; x ; $3\frac{1}{2}$

12 ; 36 ; x 4 ; 16

16 ; x ; 8 $5\frac{1}{2}$; $30\frac{1}{4}$

$20\frac{1}{4}$; $4\frac{1}{2}$ 3 ; 9

11 ; $30\frac{1}{4}$; x

33

$x^2 + 10x = 90$

$x^2 + 10x + 25 = 90 + 25$

$(x+5)^2 = 115$

$x + 5 = \sqrt{115}$ of $x + 5 = \text{-}\sqrt{115}$

$x = \text{-}5 + \sqrt{115}$ of $x = \text{-}5 - \sqrt{115}$

$x^2 + 7x + 1 = 0$

$x^2 + 7x + 1 + 11\frac{1}{4} = 11\frac{1}{4}$

$x^2 + 7x + 12\frac{1}{4} = 11\frac{1}{4}$

$(x + 3\frac{1}{2})^2 = 11\frac{1}{4} = \frac{45}{4}$

$x + 3\frac{1}{2} = \sqrt{\frac{45}{4}} = \frac{1}{2}\sqrt{45} = 1\frac{1}{2}\sqrt{5}$ of

$\qquad\qquad x + 3\frac{1}{2} = \text{-}1\frac{1}{2}\sqrt{5}$

$x = \text{-}3\frac{1}{2} + 1\frac{1}{2}\sqrt{5}$ of $x = \text{-}3\frac{1}{2} - 1\frac{1}{2}\sqrt{5}$

$x^2 + 10x + 22 = 0$

$x^2 + 10x + 22 + 3 = 3$

$x^2 + 10x + 25 = 3$

$(x + 5)^2 = 3$

$x + 5 = \sqrt{3}$ of $x + 5 = \text{-}\sqrt{3}$

$x = \text{-}5 + \sqrt{3}$ of $x = \text{-}5 - \sqrt{3}$

$x^2 - 11x = \text{-}7$

$x^2 - 11x + 30\frac{1}{4} = \text{-}7 + 30\frac{1}{4}$

$(x - 5\frac{1}{2})^2 = 23\frac{1}{4} = \frac{93}{4}$

$x - 5\frac{1}{2} = \sqrt{\frac{93}{4}} = \frac{1}{2}\sqrt{93}$ of $x - 5\frac{1}{2} = \text{-}\frac{1}{2}\sqrt{93}$

$x = 5\frac{1}{2} + \frac{1}{2}\sqrt{93}$ of $x = 5\frac{1}{2} - \frac{1}{2}\sqrt{93}$

$x^2 - 12x = \text{-}23$

$x^2 - 12x + 36 = \text{-}23 + 36$

$(x - 6)^2 = 13$

$x - 6 = \sqrt{13}$ of $x - 6 = \text{-}\sqrt{13}$

$x = 6 + \sqrt{13}$ of $x = 6 - \sqrt{13}$

$x^2 - 5x - 1 = 0$

$x^2 - 5x - 1 + 7\frac{1}{4} = 7\frac{1}{4}$

$x^2 - 5x + 6\frac{1}{4} = 7\frac{1}{4}$

$(x - 2\frac{1}{2})^2 = 7\frac{1}{4} = \frac{29}{4}$

$x - 2\frac{1}{2} = \sqrt{\frac{29}{4}} = \frac{1}{2}\sqrt{29}$ of $x - 2\frac{1}{2} = \text{-}\frac{1}{2}\sqrt{29}$

$x = 2\frac{1}{2} + \frac{1}{2}\sqrt{29}$ of $x = 2\frac{1}{2} - \frac{1}{2}\sqrt{29}$

$x^2 = x + 3$

$x^2 - x = 3$

$x^2 - x + \frac{1}{4} = 3 + \frac{1}{4}$

$(x - \frac{1}{2})^2 = 3\frac{1}{4} = \frac{13}{4}$

$x - \frac{1}{2} = \sqrt{\frac{13}{4}} = \frac{1}{2}\sqrt{13}$ of $x - \frac{1}{2} = \text{-}\frac{1}{2}\sqrt{13}$

$x = \frac{1}{2} + \frac{1}{2}\sqrt{13}$ of $x = \frac{1}{2} - \frac{1}{2}\sqrt{13}$

$x^2 - 9x - 3 = 0$

$x^2 - 9x - 3 + 23\frac{1}{4} = 23\frac{1}{4}$

$x^2 - 9x + 20\frac{1}{4} = 23\frac{1}{4}$

$(x - 4\frac{1}{2})^2 = 23\frac{1}{4} = \frac{93}{4}$

$x - 4\frac{1}{2} = \sqrt{\frac{93}{4}} = \frac{1}{2}\sqrt{93}$ of $x - 4\frac{1}{2} = \text{-}\frac{1}{2}\sqrt{93}$

$x = 4\frac{1}{2} + \frac{1}{2}\sqrt{93}$ of $x = 4\frac{1}{2} - \frac{1}{2}\sqrt{93}$

34 a $a = 2$, $b = 12$ en $c = 6$.

b $x = \frac{\text{-}12 + \sqrt{12^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2} = \frac{\text{-}12 + \sqrt{96}}{4} = \frac{\text{-}12 + 4\sqrt{6}}{4} = \text{-}3 + \sqrt{6}$

$x = \frac{\text{-}12 - \sqrt{12^2 - 4 \cdot 2 \cdot 6}}{2 \cdot 2} = \frac{\text{-}12 - \sqrt{96}}{4} = \frac{\text{-}12 - 4\sqrt{6}}{4} = \text{-}3 - \sqrt{6}$

35 Dan staat er een lineaire vergelijking.

36 $2x^2 - 3x - 35 = 0$

$\left.\begin{array}{l} a = 2 \\ b = \text{-}3 \\ c = \text{-}35 \end{array}\right\}$ $D = 9 - 4 \cdot 2 \cdot \text{-}35 = 289,$

$\qquad\qquad \sqrt{D} = 17$

$x = \frac{3 + 17}{4} = 5$ of $x = \frac{3 - 17}{4} = \text{-}3\frac{1}{2}$

$2x^2 + 4x - 1 = 0$

$\left.\begin{array}{l} a = 2 \\ b = 4 \\ c = \text{-}1 \end{array}\right\}$ $D = 16 - 4 \cdot 2 \cdot \text{-}1 = 24,$

$\qquad\quad \sqrt{D} = \sqrt{24} = 2\sqrt{6}$

$x = \frac{\text{-}4 + 2\sqrt{6}}{4} = \text{-}1 + \frac{1}{2}\sqrt{6}$ of $x = \frac{\text{-}4 - 2\sqrt{6}}{4} = \text{-}1 - \frac{1}{2}\sqrt{6}$

$7x^2 - 6x + 2 = 0$

$\left.\begin{array}{l} a = 7 \\ b = \text{-}6 \\ c = 2 \end{array}\right\}$ $D = 36 - 4 \cdot 7 \cdot 2 = \text{-}20$

$D < 0$, dus géén oplossingen

$\frac{1}{2}x^2 - 3x - 4\frac{1}{2} = 0$

$\left.\begin{array}{l} a = \frac{1}{2} \\ b = \text{-}3 \\ c = \text{-}4\frac{1}{2} \end{array}\right\}$ $D = 9 - 4 \cdot \frac{1}{2} \cdot \text{-}4\frac{1}{2} = 18,$

$\sqrt{D} = \sqrt{18} = 3\sqrt{2}$

$x = \frac{3+3\sqrt{2}}{1} = 3 + 3\sqrt{2}$ of $x = \frac{3-3\sqrt{2}}{1} = 3 - 3\sqrt{2}$

$4x = 1 + 4x^2$

$4x^2 - 4x + 1 = 0$

$\left.\begin{array}{l} a = 4 \\ b = \text{-}4 \\ c = 1 \end{array}\right\}$ $D = 16 - 4 \cdot 4 \cdot 1 = 0$

$x = \text{-}\frac{\text{-}4}{8} = \frac{1}{2}$

$(x - 3)^2 = 5 - 3x$

$x^2 - 6x + 9 = 5 - 3x$

$x^2 - 3x + 4 = 0$

$\left.\begin{array}{l} a = 1 \\ b = \text{-}3 \\ c = 4 \end{array}\right\}$ $D = 9 - 4 \cdot 1 \cdot 4 = \text{-}7$

$D < 0$, dus geen oplossingen

$5x - 3x^2 = 0$

$\left.\begin{array}{l} a = \text{-}3 \\ b = 5 \\ c = 0 \end{array}\right\}$ $D = 25 - 4 \cdot \text{-}3 \cdot 0 = 25,$

$\sqrt{D} = 5$

$x = \frac{\text{-}5+5}{\text{-}6} = 0$ of $x = \frac{\text{-}5-5}{\text{-}6} = \frac{\text{-}10}{\text{-}6} = 1\frac{2}{3}$

SUPER OPGAVEN

15 a $\frac{18}{45} = \frac{12}{y}$

$18y = 540$

$y = 30$

$O = 12 \cdot (45 - 30) = 180$

b $\frac{18}{45} = \frac{x}{y}$

$18y = 45x$

$y = 2\frac{1}{2}x$

c Breedte van de rechthoek is

$45 - y = 45 - 2\frac{1}{2}x$

$O = x \cdot (45 - 2\frac{1}{2}x) = 45x - 2\frac{1}{2}x^2$

d $45x - 2\frac{1}{2}x^2 = 0$

$2\frac{1}{2}x(18 - x) = 0$

$x = 0$ of $x = 18$

De oppervlakte is maximaal als $x = 9$.

e De oppervlakte is dan $45 \cdot 9 - 2\frac{1}{2} \cdot 9^2 = 202\frac{1}{2}$.

23 Als de top op de *y*-as ligt, dan zijn (-2,4) en (3,6) ook punten van de parabool.
Dus dan moet het een dalparabool zijn.

29 a Oppervlakte vierkant is $6 \cdot 6 = 36$,
totale oppervlakte wit is $x \cdot x + (6 - x)(6 - x) = 2x^2 - 12x + 36$,
oppervlakte oker is $36 - (2x^2 - 12x + 36) = \text{-}2x^2 + 12x$.

b $\text{-}2x^2 + 12x = 0$

$\text{-}2x(x - 6) = 0$

$x = 0$ of $x = 6$

De nulpunten zijn 0 en 6.

c Als $x = 3$, dan oppervlakte is $\text{-}2 \cdot 3^2 + 12 \cdot 3 = 18$.

30 a Stelsel:

$\begin{cases} l \cdot b = 400 \\ 2l + 2b - 4 = 96 \end{cases}$

b $2l + 2b - 4 = 96$

$2l + 2b = 100$

$l + b = 50$

$l = 50 - b$

Vergelijking:

$(50 - b) \cdot b = 400$

$50b - b^2 = 400$

$b^2 - 50b + 400 = 0$

$(b - 40)(b - 10) = 0$

$b = 40$ of $b = 10$

Als $b = 40$, dan $l = 400 : 40 = 10$.

Als $b = 10$, dan $l = 400 : 10 = 40$.

De afmetingen zijn 40 bij 10 of 10 bij 40.

29.8 EXTRA OPGAVEN

1 a $y = x^2 + x$

Nulpunten:

$x^2 + x = 0$

$x(x + 1) = 0$

$x = 0$ of $x = \text{-}1$

Snijpunt y – as:

$y = 0^2 + 0 = 0$

(0, 0)

Symmetrieas:

$x = \frac{0-1}{2} = \text{-}\frac{1}{2}$

$y = (\text{-}\frac{1}{2})^2 - \frac{1}{2} = \text{-}\frac{1}{4}$

Top($\text{-}\frac{1}{2}$, $\text{-}\frac{1}{4}$).

x	-3	-2	-1	$\text{-}\frac{1}{2}$	0	1	2
y	7	2	0	$\text{-}\frac{1}{4}$	0	2	7

$y = x^2 - 7x$

Nulpunten:

$x^2 - 7x = 0$

$x(x - 7) = 0$

$x = 0$ of $x = 7$

Snijpunt y-as:

$y = 0^2 - 7 \cdot 0 = 0$

$(0, 0)$

Symmetrieas:

$x = \frac{0+7}{2} = 3\frac{1}{2}$

$y = \left(3\frac{1}{2}\right)^2 - 7 \cdot 3\frac{1}{2} = -12\frac{1}{4}$

Top($3\frac{1}{2}$, $-12\frac{1}{4}$).

x	-1	0	1	2	3	$3\frac{1}{2}$	4	5	6	7
y	8	0	-6	-10	-12	$-12\frac{1}{4}$	-12	-10	-6	0

$y = -3x^2$

Nulpunten:

$-3x^2 = 0$

$x = 0$

Snijpunt y-as:

$y = -3 \cdot 0^2$

$(0, 0)$

Symmetrieas:

$x = \frac{0+0}{2} = 0$

$y = -3 \cdot 0^2$

Top($0, 0$).

x	-3	-2	-1	0	1	2
y	-27	-12	-3	0	-3	-12

$y = (x + 2)^2 - 3$

Nulpunten:

$(x + 2)^2 - 3 = 0$

$(x + 2)^2 = 3$

$x + 2 = \sqrt{3}$ of $x + 2 = -\sqrt{3}$

$x = -2 + \sqrt{3}$ of $x = -2 - \sqrt{3}$

Snijpunt y-as:

$y = (0 + 2)^2 - 3 = 1$

$(0, 1)$

Top($-2, -3$).

Symmetrieas:

$x = -2$

x	-5	-4	-3	-2	-1	0	1
y	6	1	-2	-3	-2	1	6

$y = -\frac{1}{2}(x - 1)^2 + 8$

Nulpunten:

$-\frac{1}{2}(x - 1)^2 + 8 = 0$

$-\frac{1}{2}(x - 1)^2 = -8$

$(x - 1)^2 = 16$

$x - 1 = 4$ of $x - 1 = -4$

$x = 5$ of $x = -3$

Snijpunt y-as:

$y = -\frac{1}{2}(0 - 1)^2 + 8 = 7\frac{1}{2}$

$(0, 7\frac{1}{2})$

Top($1, 8$).

Symmetrieas:

$x = \frac{-3+5}{2} = 1$

x	-3	-2	-1	0	1	2	3	4	5	6
y	0	$3\frac{1}{2}$	6	$7\frac{1}{2}$	8	$7\frac{1}{2}$	6	$3\frac{1}{2}$	0	$-4\frac{1}{2}$
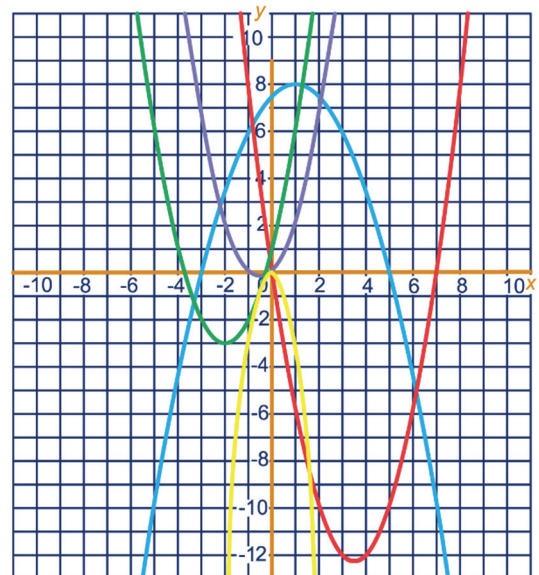
2   $y = x^2 + 12x$

Nulpunten:

$x^2 + 12x = 0$

$x(x + 12) = 0$

$x = 0$ of $x = -12$

Symmetrieas:

$x = \frac{0-12}{2} = -6$

$y = (-6)^2 + 12 \cdot -6 = -36$

Top($-6, -36$).

$y = 2x^2 - 5x$

Nulpunten:

$2x^2 - 5x = 0$

$2x(x - 2\frac{1}{2}) = 0$

$x = 0$ of $x = 2\frac{1}{2}$

Symmetrieas:

$x = \frac{0+2\frac{1}{2}}{2} = 1\frac{1}{4}$

$y = 2 \cdot (1\frac{1}{4})^2 - 5 \cdot 1\frac{1}{4} = \text{-}3\frac{1}{8}$

Top($1\frac{1}{4}$, -$3\frac{1}{8}$).

$y = x^2 + 3x + 2$

Nulpunten:

$x^2 + 3x + 2 = 0$

$(x + 1)(x + 2) = 0$

$x = \text{-}1$ of $x = \text{-}2$

Symmetrieas:

$x = \frac{\text{-}1-2}{2} = \text{-}1\frac{1}{2}$

$y = (\text{-}1\frac{1}{2})^2 + 3 \cdot \text{-}1\frac{1}{2} + 2 = \text{-}\frac{1}{4}$

Top(-$1\frac{1}{2}$, -$\frac{1}{4}$).

$y = \text{-}x^2 + 4x + 6$

$\text{-}x^2 + 4x + 6 = 6$

$\text{-}x^2 + 4x = 0$

$\text{-}x(x - 4) = 0$

$x = 0$ of $x = 4$

Symmetrieas:

$x = \frac{0+4}{2} = 2$

$y = \text{-}2^2 + 4 \cdot 2 + 6 = 10$

Top(2, 10).

3 a $y = cx^2$

    $3 = c \cdot 4^2$ (invullen het punt (4,3))

    $3 = 16c$

    $\frac{3}{16} = c$

    Vergelijking parabool: $y = \frac{3}{16} x^2$

b $x = 3$ of $x = \text{-}3 \Rightarrow y = \frac{3}{16} \cdot 3^2 = 1\frac{11}{16}$

    Dus ($3, 1\frac{11}{16}$) en (-$3, 1\frac{11}{16}$).

4   100 ; 10

    18; 81 ; x

    $42\frac{1}{4}$; $6\frac{1}{2}$

    5 ; $6\frac{1}{4}$; x

    6 ; 36

    1 ; 1

5 $14 = x(x - 5)$

    $x^2 - 5x - 14 = 0$

    $(x - 7)(x + 2) = 0$

    $x = 7$ of $x = \text{-}2$

    $2x^2 + x = 5x + 8$

    $2x^2 - 4x - 8 = 0$

    $x^2 - 2x - 4 = 0$

    $x^2 - 2x - 4 + 5 = 5$

    $x^2 - 2x + 1 = 5$

    $(x - 1)^2 = 5$

    $x - 1 = \sqrt{5}$ of $x - 1 = \text{-}\sqrt{5}$

    $x = 1 + \sqrt{5}$ of $x = 1 - \sqrt{5}$

    $25 = 4(x + 1)^2$

    $(x + 1)^2 = 6\frac{1}{4}$

    $x + 1 = 2\frac{1}{2}$ of $x + 1 = \text{-}2\frac{1}{2}$

    $x = 1\frac{1}{2}$ of $x = \text{-}3\frac{1}{2}$

    $(x + 1)^2 + (x + 3)^2 = 4x^2$

    $2x^2 + 8x + 10 = 4x^2$

    $2x^2 - 8x - 10 = 0$

    $x^2 - 4x - 5 = 0$

    $(x - 5)(x + 1) = 0$

    $x = 5$ of $x = \text{-}1$

    $x^2 - 3x = 2x^2 + x + 1$

    $x^2 + 4x + 1 = 0$

    $x^2 + 4x + 1 + 3 = 3$

    $x^2 + 4x + 4 = 3$

    $(x + 2)^2 = 3$

    $x + 2 = \sqrt{3}$ of $x + 2 = \text{-}\sqrt{3}$

    $x = \text{-}2 + \sqrt{3}$ of $x = \text{-}2 - \sqrt{3}$

    $x^2 + 5x + 3 = 0$

    $x^2 + 5x + 3 + 3\frac{1}{4} = 3\frac{1}{4}$

    $x^2 + 5x + 6\frac{1}{4} = 3\frac{1}{4}$

    $(x + 2\frac{1}{2})^2 = 3\frac{1}{4} = \frac{13}{4}$

    $x + 2\frac{1}{2} = \sqrt{\frac{13}{4}} = \frac{1}{2}\sqrt{13}$ of $x + 2\frac{1}{2} = \text{-}\frac{1}{2}\sqrt{13}$

    $x = \text{-}2\frac{1}{2} + \frac{1}{2}\sqrt{13}$ of $x = \text{-}2\frac{1}{2} - \frac{1}{2}\sqrt{13}$

6 a Oppervlakte grasveld is $4 \cdot 4 = 16$,
oppervlakte border is $3 \cdot 4x + 2 \cdot x^2 = 12x + 2x^2$.

$$12x + 2x^2 = 16$$

$$x^2 + 6x - 8 = 0$$

$$x^2 + 6x - 8 + 17 = 17$$

$$x^2 + 6x + 9 = 17$$

$$(x + 3)^2 = 17$$

$$x + 3 = \sqrt{17} \quad \text{of} \quad x + 3 = \text{-}\sqrt{17}$$

$$x = \text{-}3 + \sqrt{17} \quad \text{of} \quad x = \text{-}3 - \sqrt{17}$$

Dus $x = \text{-}3 + \sqrt{17}$ m, omdat $x > 0$ moet zijn.

b $12x + 2x^2 = 2 \cdot 16$

$$x^2 + 6x - 16 = 0$$

$$(x + 8)(x - 2) = 0$$

$$x = \text{-}8 \quad \text{of} \quad x = 2$$

Dus $x = 2$ m, omdat $x > 0$ moet zijn.

7 $3x^2 + 10x + 3 = 0$

$\left.\begin{array}{l} a = 3 \\ b = 10 \\ c = 3 \end{array}\right\}$ $D = 100 - 4 \cdot 3 \cdot 3 = 64,$ $\sqrt{D} = 8$

$$x = \frac{\text{-}10 + 8}{6} = \text{-}\tfrac{1}{3} \quad \text{of} \quad x = \frac{\text{-}10 - 8}{6} = \text{-}3$$

$$2x^2 = 5x - 3$$

$$2x^2 - 5x + 3 = 0$$

$\left.\begin{array}{l} a = 2 \\ b = \text{-}5 \\ c = 3 \end{array}\right\}$ $D = 25 - 4 \cdot 2 \cdot 3 = 1,$ $\sqrt{D} = 1$

$$x = \frac{5 + 1}{4} = 1\tfrac{1}{2} \quad \text{of} \quad x = \frac{5 - 1}{2} = 2$$

$$x^2 - 8x = \text{-}22$$

$$x^2 - 8x + 22 = 0$$

$\left.\begin{array}{l} a = 1 \\ b = \text{-}8 \\ c = 22 \end{array}\right\}$ $D = 64 - 4 \cdot 1 \cdot 22 = \text{-}24$

$D < 0$, dus geen oplossingen

$$\text{-}5x^2 + 4x - \tfrac{4}{5} = 0$$

$\left.\begin{array}{l} a = \text{-}5 \\ b = 4 \\ c = \text{-}\tfrac{4}{5} \end{array}\right\}$ $D = 16 - 4 \cdot \text{-}5 \cdot \text{-}\tfrac{4}{5} = 0$

$$x = \text{-}\frac{4}{\text{-}10} = \tfrac{2}{5}$$

8 a Oppervlakte driehoek is $\tfrac{1}{2} \cdot x(8 - x) = 4x - \tfrac{1}{2}x^2$.

b $\tfrac{1}{4}$ deel ; $\tfrac{1}{4} \cdot 8 \cdot 8 = 16$

c $2 \cdot x(8 - x) = 16$

$$2x^2 - 16x + 16 = 0$$

$$x^2 - 8x + 8 = 0$$

$\left.\begin{array}{l} a = 1 \\ b = \text{-}8 \\ c = 8 \end{array}\right\}$ $D = 64 - 4 \cdot 1 \cdot 8 = 32,$ $\sqrt{D} = \sqrt{32} = 4\sqrt{2}$

$$x = \frac{8 + 4\sqrt{2}}{2} = 4 + 2\sqrt{2} \text{ cm} \quad \text{of} \quad x = 4 - 2\sqrt{2} \text{ cm}$$

9 a hoogte $= x$, breedte $= x + 3$, lengte $= x + 4$

b oppervlakte $=$
$$2(x(x + 4) + x(x + 3) + (x + 4)(x + 3)) =$$
$$2(3x^2 + 14x + 12) = 6x^2 + 28x + 24$$

$$6x^2 + 28x + 24 = 162$$

$$6x^2 + 28x - 138 = 0$$

$\left.\begin{array}{l} a = 6 \\ b = 28 \\ c = \text{-}138 \end{array}\right\}$ $D = 784 - 4 \cdot 6 \cdot \text{-}138 = 4096,$ $\sqrt{D} = 64$

$$x = \frac{\text{-}28 + 64}{12} = 3 \quad \text{of} \quad x = \frac{\text{-}28 - 64}{12} = \text{-}7\tfrac{2}{3}$$

Alleen $x = 3$ voldoet, omdat $x > 0$ moet zijn.
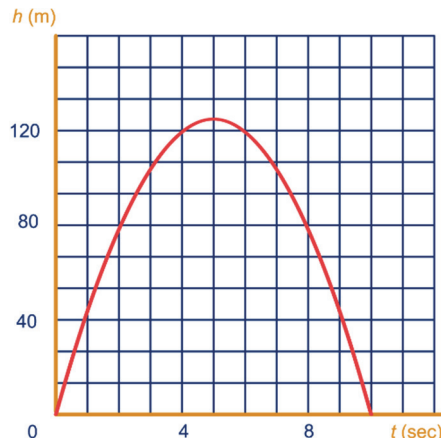
10 a $50t - 5t^2 = 0$

$$5t(10 - t) = 0$$

$$t = 0 \quad \text{of} \quad t = 10$$

Dus de vlucht duurt $10 - 0 = 10$ sec.

b Maximale hoogte wordt bereikt na 5 sec.,
$h = 50 \cdot 5 - 5 \cdot 5^2 = 250 - 125 = 125$ m.

c

d $50t - 5t^2 > 113{,}75$

$$0 > 5t^2 - 50t + 113{,}75$$

$$t^2 - 10t + 22{,}75 < 0$$

$$(t - 3{,}5)(t - 6{,}5) < 0$$

$$3{,}5 < t < 6{,}5$$

Dus tussen de 3,5 en 6,5 sec. is de hoogte van de vuurpijl meer dan 113,75 m.