H28 VIERKANTSVERGELIJKINGEN VWO

28.0 INTRO

1 -

2 -

28.1 TERUGBLIKKEN

3
a $x = 3\frac{1}{2}$
b $2x + 7 = 4x + 1$
$7 = 2x + 1$
$6 = 2x$
$x = 3$
c $x = 4$ of $x = -4$
d $x + 6 = 4$ of $x + 6 = -4$
$x = -2$ of $x = -10$
e Er is geen oplossing, want het kwadraat van een getal kan niet negatief zijn.
f $x = \sqrt{7}$ of $x = -\sqrt{7}$
g $x + 5 = \sqrt{7}$ of $x + 5 = -\sqrt{7}$
$x = -5 + \sqrt{7}$ of $x = -5 - \sqrt{7}$
h $x^2 - x = 0$
$x(x - 1) = 0$
$x = 0$ of $x = 1$
i $x^2 + x = 0$
$x(x + 1) = 0$
$x = 0$ of $x = -1$
j $x^2 - 2x = 0$
$x(x - 2) = 0$
$x = 0$ of $x = 2$
k $x = 7$
l $x + 1 = 7$
$x = 6$
m $\frac{2}{2x} = \frac{2}{7}$
$2x = 7$
$x = 3\frac{1}{2}$
n $x = \frac{1}{7}$
o $\frac{1}{x} = 3\frac{1}{2}$
$x = \frac{1}{3\frac{1}{2}} = \frac{2}{7}$
p $\frac{4}{2x} = \frac{4}{7}$
$2x = 7$
$x = 3\frac{1}{2}$
q $x = 49$
r Er is geen oplossing, want de wortel van een getal kan niet negatief zijn.
s $x + 1 = 49$
$x = 48$
t $x^2 = \frac{1}{7}$
$x = \sqrt{\frac{1}{7}} = \frac{1}{7}\sqrt{7}$ of $x = -\sqrt{\frac{1}{7}} = -\frac{1}{7}\sqrt{7}$

4
a $x^2 + 12x + 36 = 16$
$x^2 + 12x + 20 = 0$
b $x^2 + 12x + 20 = (x + 2)(x + 10)$
c $x = -2$ of $x = -10$

5
a

$1 \cdot -24 = -24$	$-1 \cdot 24 = -24$
$2 \cdot -12 = -24$	$-2 \cdot 12 = -24$
$3 \cdot -8 = -24$	$-3 \cdot 8 = -24$
$4 \cdot -6 = -24$	$-4 \cdot 6 = -24$

b $x^2 + 5x - 24 = (x + 8)(x - 3)$
$x^2 + 5x - 24 = 0$
$(x + 8)(x - 3) = 0$
$x = -8$ of $x = 3$

6
a $(x + 4)(x - 1) = 0$, dus $x = -4$ of $x = 1$
b $(x + 6)(x - 4) = 0$, dus $x = -6$ of $x = 4$
c $(x + 6)(x + 4) = 0$, dus $x = -6$ of $x = -4$
d $x(x + 4) = 0$, dus $x = 0$ of $x = -4$

7
a $x^2 + 2x - 48 = 0$
$(x + 8)(x - 6) = 0$
$x = -8$ of $x = 6$
b $x^2 - 2x = 0$
$x(x - 2) = 0$
$x = 0$ of $x = 2$
c $2(x^2 + 2x - 3) = 0$
$2(x + 3)(x - 1) = 0$
$x = -3$ of $x = 1$
d $-x^2 + 4 = -60$
$x^2 = 64$
$x = 8$ of $x = -8$
e $x^2 = 5x + 50$
$x^2 - 5x - 50 = 0$
$(x - 10)(x + 5) = 0$
$x = 10$ of $x = -5$
f $x^2 - x + 3x - 3 = 117$
$x^2 + 2x - 120 = 0$
$(x + 12)(x - 10) = 0$
$x = -12$ of $x = 10$
g $x(x^2 + 2x - 3) = 0$
$x(x + 3)(x - 1) = 0$
$x = 0$ of $x = -3$ of $x = 1$
h $x^5 - 4x^4 = 0$
$x^4(x - 4) = 0$
$x = 0$ of $x = 4$
i $x^2 + 2x + 1 = x + 3$
$x^2 + x - 2 = 0$
$(x + 2)(x - 1) = 0$
$x = -2$ of $x = 1$
j $x^2(x^2 + 4x + 4) = 0$
$x^2(x + 2)^2 = 0$
$x = 0$ of $x = -2$

28.2 KWADRAATAFSPLITSEN

8
a $x^2 + 3x + 3x = x^2 + 6x$
b $x^2 + 6x = 7$
$x^2 + 6x - 7 = 0$
$(x + 7)(x - 1) = 0$
$x = -7$ of $x = 1$
Dus $x = 1$, want x kan niet negatief zijn.

c $x^2 + 6x = 16$
$x^2 + 6x - 16 = 0$
$(x + 8)(x - 2) = 0$
$x = -8$ of $x = 2$
Dus $x = 2$, want x kan niet negatief zijn.

d $(-3 + \sqrt{19})^2 + 6(-3 + \sqrt{19}) =$
$9 - 6\sqrt{19} + 19 - 18 + 6\sqrt{19} = 10$, klopt.

e 9

f zijde = $x + 3$; oppervlakte = $(x + 3)^2$

g $x^2 + 6x = (x + 3)^2 - 9$

h $x^2 + 6x = 11$
$(x + 3)^2 - 9 = 11$
$(x + 3)^2 = 20$
$x + 3 = \sqrt{20}$ of $x + 3 = -\sqrt{20}$
$x = -3 + 2\sqrt{5}$ of $x = -3 - 2\sqrt{5}$

9 a $x + 5$

b 25

c $x = -5 + \sqrt{5}$ of $x = -5 - \sqrt{5}$

d $(x + 5)^2 = 37$

e $x = -5 + \sqrt{37}$ of $x = -5 - \sqrt{37}$

10 a $x^2 + 12x = (x + 6)^2 - 36$

b $x^2 + 12x = 4$
$(x + 6)^2 - 36 = 4$
$(x + 6)^2 = 40$
$x + 6 = \sqrt{40}$ of $x + 6 = -\sqrt{40}$
$x = -6 + 2\sqrt{10}$ of $x = -6 - 2\sqrt{10}$

c $x^2 + 12x + 4 = 0$
$(x + 6)^2 - 36 + 4 = 0$
$(x + 6)^2 = 32$
$x + 6 = \sqrt{32}$ of $x + 6 = -\sqrt{32}$
$x = -6 + 4\sqrt{2}$ of $x = -6 - 4\sqrt{2}$

11 a $x^2 - 20x = (x - 10)^2 - 100$

b $x^2 - 7x = (x - 3\frac{1}{2})^2 - 12\frac{1}{4}$

c $x^2 - 8x = (x - 4)^2 - 16$

d $x^2 + 11x = (x + 5\frac{1}{2})^2 - 30\frac{1}{4}$

e $x^2 - 21x = (x - 10\frac{1}{2})^2 - 110\frac{1}{4}$

f $x^2 + x = (x + \frac{1}{2})^2 - \frac{1}{4}$

g $x^2 - x = (x - \frac{1}{2})^2 - \frac{1}{4}$

28.3 VIERKANTSVERGELIJKINGEN OPLOSSEN

12 a $-x^2 + 3x = 4x - 5$
$x^2 - 3x = -4x + 5$
$x^2 + x = 5$
$(x + \frac{1}{2})^2 - \frac{1}{4} = 5$
$(x + \frac{1}{2})^2 = 5\frac{1}{4} = \frac{21}{4}$
$x = -\frac{1}{2} + \frac{1}{2}\sqrt{21}$ of $x = -\frac{1}{2} - \frac{1}{2}\sqrt{21}$
(NB: $\sqrt{\frac{21}{4}} = \frac{1}{2}\sqrt{21}$)

b $2x^2 = 4x + 6$
$x^2 = 2x + 3$
$x^2 - 2x = 3$
$(x - 1)^2 - 1 = 3$
$(x - 1)^2 = 4$
$x - 1 = 2$ of $x - 1 = -2$
$x = 3$ of $x = -1$

Handiger:
$x^2 - 2x - 3 = 0$
$(x - 3)(x + 1) = 0$
$x = 3$ of $x = -1$

c $(x + 1)^2 = -(x + 2) + 7$
$x^2 + 2x + 1 = -x - 2 + 7$
$x^2 + 3x - 4 = 0$
$(x + 4)(x - 1) = 0$
$x = -4$ of $x = 1$

d $x^2 + 5x + 3 = 0$
$(x + 2\frac{1}{2})^2 - 6\frac{1}{4} + 3 = 0$
$(x + 2\frac{1}{2})^2 = \frac{13}{4}$
$x + 2\frac{1}{2} = \sqrt{\frac{13}{4}}$ of $x + 2\frac{1}{2} = -\sqrt{\frac{13}{4}}$
$x = -2\frac{1}{2} + \frac{1}{2}\sqrt{13}$ of $x = -2\frac{1}{2} - \frac{1}{2}\sqrt{13}$

e $3x^2 + 6x + 9 = 0$
$x^2 + 2x + 3 = 0$
$(x + 1)^2 - 1 + 3 = 0$
$(x + 1)^2 = -2$
Er zijn geen oplossingen.

f $-2x^2 - 4x = 20$
$-x^2 - 2x - 10 = 0$
$x^2 + 2x + 10 = 0$
$(x + 1)^2 - 1 + 10 = 0$
$(x + 1)^2 = -9$
Er zijn geen oplossingen.

g $(2x)^2 = 4x - 1$
$4x^2 - 4x + 1 = 0$
$x^2 - x + \frac{1}{4} = 0$
$(x - \frac{1}{2})^2 = 0$
$x = \frac{1}{2}$

28.4 KRUISLINGS VERMENIGVULDIGEN

13 a $2x + 1$

b 3

14 a $2x + 1 = 3x$
$x = 1$

b Beide kanten met $x + 1$ vermenigvuldigen geeft:
$2x + 3 = 4(x + 1)$
$2x + 3 = 4x + 4$
$-1 = 2x$
$x = -\frac{1}{2}$

c $x^2 + 1 = 2x^2 - 4$
$x^2 = 5$
$x = \sqrt{5}$ of $x = -\sqrt{5}$

d $x - 1 = x^2 + 3x$
$x^2 + 2x + 1 = 0$
$(x + 1)^2 = 0$
$x = -1$

15 a $4(x - 1) = 2(x + 1)$
$4x - 4 = 2x + 2$
$2x = 6$
$x = 3$

b $x(x - 1) = (2x + 1)(x + 1)$
$x^2 - x = 2x^2 + 3x + 1$
$x^2 + 4x + 1 = 0$
$(x + 2)^2 = 3$
$x + 2 = \sqrt{3}$ of $x + 2 = -\sqrt{3}$
$x = -2 + \sqrt{3}$ of $x = -2 - \sqrt{3}$

c $(x + 1)^2 = 4x^2$
$x + 1 = 2x$ of $x + 1 = -2x$
$x = 1$ of $3x = -1$
$x = 1$ of $x = -\frac{1}{3}$

d $3(1 - x) = 2x - 2$
$3 - 3x = 2x - 2$
$5x = 5$
$x = 1$, maar let op, zie het antwoord bij **e**!!

e De linkerkant wordt dan bijvoorbeeld $\frac{3}{0}$ en dit heeft geen betekenis.

f $x = -\frac{1}{2}$ en $x = 1$; $x = 0$ en $x = -1$

g De waarden 0 en 1.

h $x = x^2 - x$
$x^2 - 2x = 0$
$x(x - 2) = 0$
$x = 0$ of $x = 2$
Maar $x = 0$ maakt noemers 0, dus de enige oplossing is $x = 2$.

16 a groot: x
klein: 1
hele lijnstuk: $x + 1$
Dus groot : klein = hele lijnstuk : groot wordt dan $x : 1 = (x + 1) : x$. Dus $\frac{x}{1} = \frac{x+1}{x}$.

b $x^2 = x + 1$
$x^2 - x - 1 = 0$
$(x - \frac{1}{2})^2 - \frac{1}{4} - 1 = 0$
$(x - \frac{1}{2})^2 = \frac{5}{4}$
$x - \frac{1}{2} = \sqrt{\frac{5}{4}} = \frac{1}{2}\sqrt{5}$ of $x - \frac{1}{2} = -\sqrt{\frac{5}{4}} = -\frac{1}{2}\sqrt{5}$
$x = \frac{1}{2} + \frac{1}{2}\sqrt{5}$ of $x = \frac{1}{2} - \frac{1}{2}\sqrt{5}$
Dus het gulden getal is $\frac{1}{2} + \frac{1}{2}\sqrt{5} = \frac{1+\sqrt{5}}{2}$.

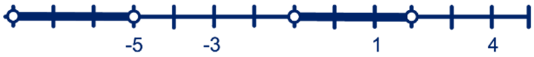
28.5 CIRKELS

17 9,6 – 6,7 = 2,9 hm
11,4 – 9,6 = 1,8 hm

18 a -3 – -5 = 2 ; 1 – -3 = 4 ; 4 – 1 = 3

b -3 – 4 = -7 en -3 + 4 = 1

c



d $7 - \sqrt{5}$; $7 + \sqrt{5}$; $7 - \sqrt{5}$

e Als $x > 4$, dan is de afstand van x tot 4: $x - 4$.
Als $x < 4$, dan is de afstand van x tot 4: $4 - x$.

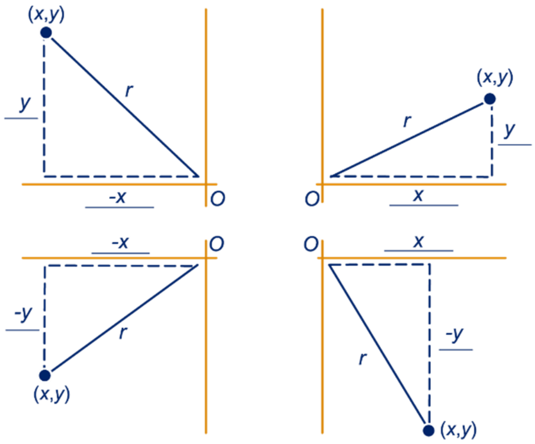
f als $x > 0$, dan is de afstand van x tot 0: x.
als $x < 0$, dan is de afstand van x tot 0: $-x$.

19 a $AO = \sqrt{7^2 + 1^2} = \sqrt{50} = 5\sqrt{2}$

b $OB = \sqrt{5^2 + 5^2} = 5\sqrt{2}$
$OC = \sqrt{1^2 + 7^2} = 5\sqrt{2}$
$OD = \sqrt{5^2 + 5^2} = 5\sqrt{2}$
$OE = \sqrt{5^2 + 5^2} = 5\sqrt{2}$

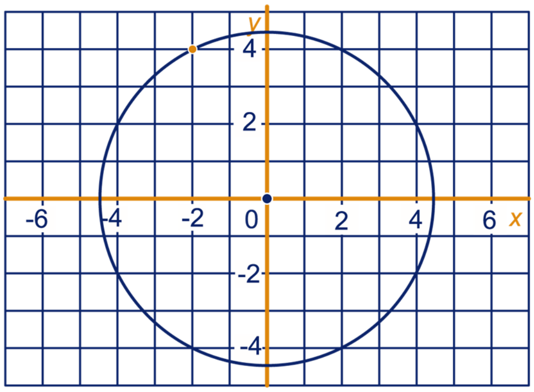
c Een cirkel met middelpunt $O(0,0)$ en straal $5\sqrt{2}$.

20 a



b $(-x)^2 = x^2$ en $(-y)^2 = y^2$

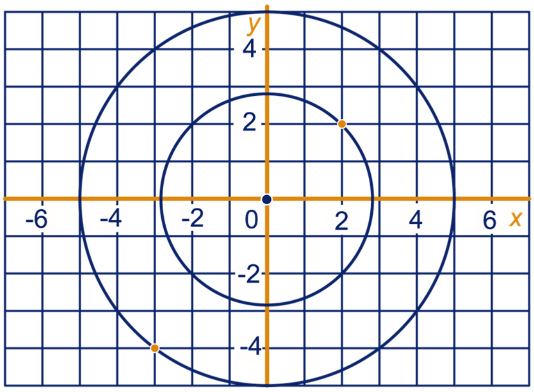
21 ac



b $(-2)^2 + 4^2 = 20$, klopt.
$r = \sqrt{20} = 2\sqrt{5}$

d Dan $y = 0$, dus $x^2 = 20$, dus
$x = \sqrt{20} = 2\sqrt{5}$ of $x = -\sqrt{20} = -2\sqrt{5}$.
Dus $(2\sqrt{5}, 0)$ en $(-2\sqrt{5}, 0)$.

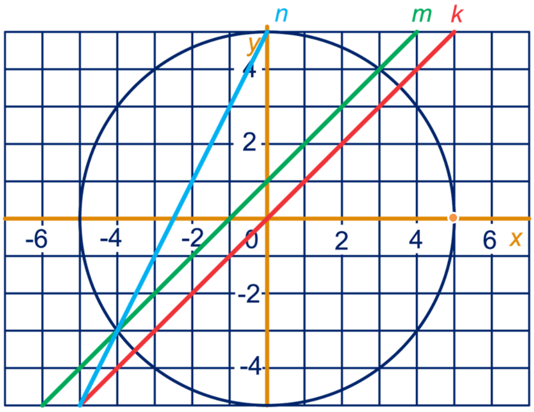
22 a



b $x^2 + y^2 = 2^2 + 2^2 = 8$ en $x^2 + y^2 = 3^2 + 4^2 = 25$

23 a $r = \sqrt{25} = 5$

bdfh



c (3,4) , (-3,4) , (-4,3) , (5,0) , (0,-5) , enzovoort.

e $x^2 + x^2 = 25$, oftewel $2x^2 = 25$

$x^2 = \frac{25}{2} = \frac{50}{4}$

$x = \sqrt{\frac{50}{4}} = \sqrt{\frac{1}{4}} \cdot \sqrt{25} \cdot \sqrt{2} = 2\frac{1}{2}\sqrt{2}$ of $x = -2\frac{1}{2}\sqrt{2}$

Snijpunten ($2\frac{1}{2}\sqrt{2}$, $2\frac{1}{2}\sqrt{2}$) en ($-2\frac{1}{2}\sqrt{2}$, $-2\frac{1}{2}\sqrt{2}$).

g $a^2 + a^2 + 2a + 1 = 25$

$2a^2 + 2a - 24 = 0$

$a^2 + a - 12 = 0$

$(a + 4)(a - 3) = 0$

$a = -4$ of $a = 3$

Als $a = -4$, dan $y = a + 1 = -4 + 1 = -3$.

Als $a = 3$, dan $y = a + 1 = 3 + 1 = 4$.

Snijpunten (-4,-3) en (3,4).

i Snijpunt is $(a, 2a + 5)$

$a^2 + (2a + 5)^2 = 25$

$a^2 + 4a^2 + 20a + 25 = 25$

$5a^2 + 20a = 0$

$5a(a + 4) = 0$

$a = 0$ of $a = -4$

Als $a = 0$, dan $y = 2a + 5 = 0 + 5 = 5$.

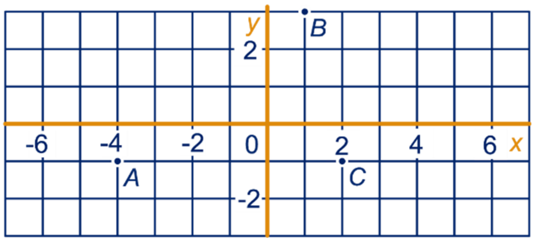
Als $a = -4$, dan $y = 2a + 5 = -8 + 5 = -3$.

Snijpunten (0,5) en (-4,-3).

j (0,0)

k Niet één.

24 a

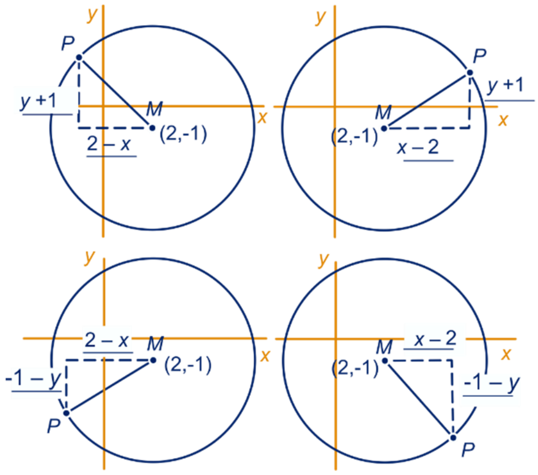


b $AC = 2 - -4 = 6$; $BC = 3 - -1 = 4$

c 87 + 101 = 188

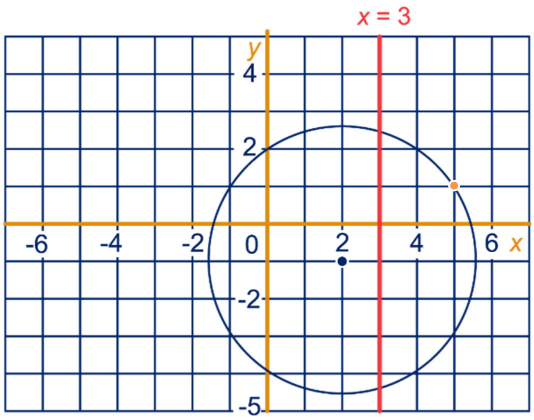
d $a - b$; $b - a$

25 a



b Omdat tegengestelde getallen hetzelfde kwadraat hebben.

c



d $(3 - 2)^2 + (y + 1)^2 = 13$

$(y + 1)^2 = 12$

$y + 1 = \sqrt{12} = 2\sqrt{3}$ of $y + 1 = -\sqrt{12} = -2\sqrt{3}$

$y = -1 + 2\sqrt{3}$ of $y = -1 + 2\sqrt{3}$

Snijpunten ($3, -1 + 2\sqrt{3}$) en ($3, -1 + 2\sqrt{3}$).

e $y = 0$, dus $(x - 2)^2 + 1 = 13$

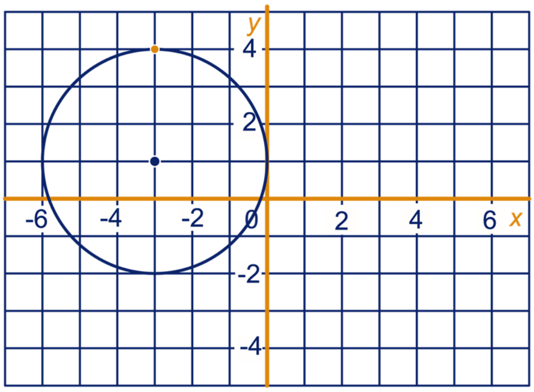
$(x - 2)^2 = 12$

$x - 2 = \sqrt{12} = 2\sqrt{3}$ of $x - 2 = -2\sqrt{3}$

$x = 2 + 2\sqrt{3}$ of $x = 2 - 2\sqrt{3}$

Snijpunten ($2 + 2\sqrt{3}$,0) en ($2 - 2\sqrt{3}$,0).

26 a



b -

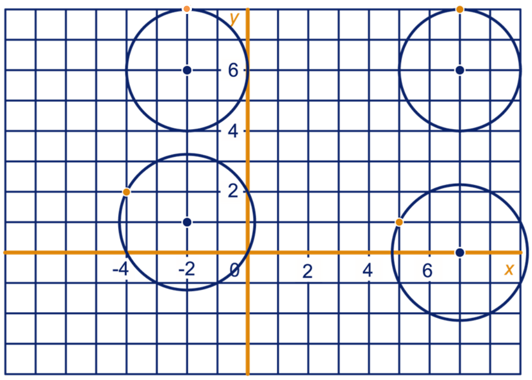
c -

d $x + 3$; $1 - y$

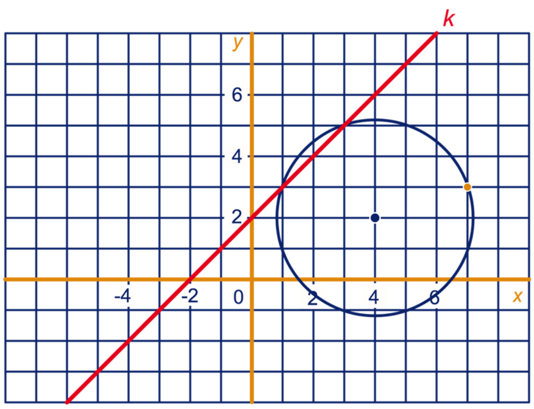
e $(x + 3)^2 + (1 - y)^2 = 9$ en dat is gelijk aan
$(x + 3)^2 + (y - 1)^2 = 9$

27 $M_{C_1}(3,3)$ en $r^2 = 3^2 = 9 \Rightarrow$
$(x - 3)^2 + (y - 3)^2 = 9$
$M_{C_2}(-4,5)$ en $r^2 = 1^2 + 2^2 = 5 \Rightarrow$
$(x + 4)^2 + (y - 5)^2 = 5$
$M_{C_3}(-3,-2)$ en $r^2 = 3^2 + 2^2 = 13 \Rightarrow$
$(x + 3)^2 + (y + 2)^2 = 13$
$M_{C_4}(2,-3)$ en $r^2 = 2^2 + 2^2 = 8 \Rightarrow$
$(x - 2)^2 + (y + 3)^2 = 8$

28



29 a



b $(x - 4)^2 + (x + 2 - 2)^2 = 10$
$x^2 - 8x + 16 + x^2 = 10$
$2x^2 - 8x + 6 = 0$
$x^2 - 4x + 3 = 0$
$(x - 1)(x - 3) = 0$
$x = 1$ of $x = 3$
Als $x = 1$, dan $y = 1 + 2 = 3$.
Als $x = 3$, dan $y = 3 + 2 = 5$.
Snijpunten: (1,3) en (3,5).

c $x^2 - 8x + 16 + y^2 - 4y + 4 = 10$
$x^2 + y^2 - 8x - 4y + 10 = 0$

30 a $(x + 5)^2 - 25$; $(y - 6)^2 - 36$

b $(x + 5)^2 - 25 + (y - 6)^2 - 36 = 39$
$(x + 5)^2 + (y - 6)^2 = 39 + 25 + 36 = 100$

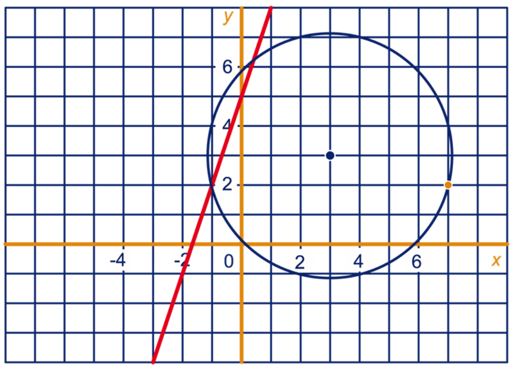
c Middelpunt (-5,6) en straal $\sqrt{100} = 10$.

31 $x^2 + 4x = (x + 2)^2 - 4$ en
$y^2 - 5y = (y - 2\frac{1}{2})^2 - 6\frac{1}{4}$
Dus:
$x^2 + y^2 + 4x - 5y + 8 = 0$
$(x + 2)^2 - 4 + (y - 2\frac{1}{2})^2 - 6\frac{1}{4} + 8 = 0$
$(x + 2)^2 + (y - 2\frac{1}{2})^2 = 4 + 6\frac{1}{4} - 8 = 2\frac{1}{4}$
Middelpunt $(-2, 2\frac{1}{2})$ en straal $\sqrt{2\frac{1}{4}} = 1\frac{1}{2}$.

28.6 GEMENGDE OPGAVEN

32 a $(x - 3)^2 - 9 + (y - 3)^2 - 9 + 1 = 0$
$(x - 3)^2 + (y - 3)^2 = 17$
Middelpunt (3,3) en straal $\sqrt{17}$.

bc



d $(x - 3)^2 + (3x + 5 - 3)^2 = 17$
$(x - 3)^2 + (3x + 2)^2 = 17$
$x^2 - 6x + 9 + 9x^2 + 12x + 4 = 17$
$10x^2 + 6x + 13 = 17$
$x^2 + \frac{3}{5}x - \frac{2}{5} = 0$
$(x + \frac{3}{10})^2 = \frac{49}{100}$
$x + \frac{3}{10} = \sqrt{\frac{49}{100}} = \frac{7}{10}$ of $x + \frac{3}{10} = -\sqrt{\frac{49}{100}} = -\frac{7}{10}$
$x = \frac{4}{10} = \frac{2}{5}$ of $x = -1$
Als $x = \frac{2}{5}$, dan $y = 3 \cdot \frac{2}{5} + 5 = 6\frac{1}{5}$.
Als $x = -1$, dan $y = 3 \cdot -1 + 5 = 2$.
Snijpunten $(\frac{2}{5}, 6\frac{1}{5})$ en (-1,2).

33 a $(60 - 2x)^2 = 500$
$60 - 2x = 10\sqrt{5}$ of $60 - 2x = -10\sqrt{5}$
$x = 30 - 5\sqrt{5}$ of $x = 30 + 5\sqrt{5}$
Maar $0 < x < 30$, dus alleen $x = 30 - 5\sqrt{5}$ voldoet.

b $(60 - 2x)^2 = 4 \cdot x \cdot (60 - 2x)$
$3600 - 240x + 4x^2 = 240x - 8x^2$
$12x^2 - 480x + 3600 = 0$
$x^2 - 40x + 300 = 0$
$(x - 10)(x - 30) = 0$
$x = 10$ of $x = 30$
Maar $0 < x < 30$, dus alleen $x = 10$ voldoet.

34 a $2(x + 3) = x(x + 1)$
$2x + 6 = x^2 + x$
$x^2 - x - 6 = 0$
$(x - 3)(x + 2) = 0$
$x = 3$ of $x = -2$
Geen van beide oplossingen maken noemers 0, dus de oplossingen zijn $x = 3$ en $x = -2$.

b $2(x + 1) = 1(x^2 + x)$
$2x + 2 = x^2 + x$
$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$
$x = 2$ of $x = -1$
$x = -1$ maakt noemers 0, dus de enige oplossing is $x = 2$.

35 a $C = (6 - 2 \cdot 1\frac{1}{2}) \cdot 1\frac{1}{2} \cdot 10 = 45$

b $C = (6 - 2x) \cdot x \cdot 10 = 60x - 20x^2$

c $60x - 20x^2 = 20$
$-2x^2 + 6x - 2 = 0$
$x^2 - 3x + 1 = 0$
$(x - 1\frac{1}{2})^2 - 2\frac{1}{4} + 1 = 0$
$(x - 1\frac{1}{2})^2 = 1\frac{1}{4} = \frac{5}{4}$
$x = 1\frac{1}{2} + \frac{1}{2}\sqrt{5}$ of $x = 1\frac{1}{2} - \frac{1}{2}\sqrt{5}$
Allebei de oplossingen voldoen.

36 a In de kleine: $\frac{x-3}{x} = \tan(\alpha)$
In de grote: $\frac{8}{x+7} = \tan(\alpha)$

b $\frac{x-3}{x} = \frac{8}{x+7}$
$(x - 3)(x + 7) = 8x$
$x^2 + 4x - 21 = 8x$
$x^2 - 4x - 21 = 0$
$(x - 7)(x + 3) = 0$
$x = 7$ of $x = -3$
Maar $x > 0$, dus alleen $x = 7$ voldoet.

37 a $36 - (6 - x)^2 - x^2 = 36 - (36 - 12x + x^2) - x^2 = 36 - 36 + 12x - x^2 - x^2 = 12x - 2x^2$

b $12x - 2x^2 = 2$
$0 = 2x^2 - 12x + 2$
$x^2 - 6x + 1 = 0$
$(x - 3)^2 - 9 + 1 = 0$
$(x - 3)^2 = 8$
$x - 3 = \sqrt{8} = 2\sqrt{2}$ of $x - 3 = -2\sqrt{2}$
$x = 3 + 2\sqrt{2}$ of $x = 3 - 2\sqrt{2}$
Maar $0 < x < 3$, dus alleen $x = 3 - 2\sqrt{2}$ voldoet.

38 a Border: $2x^2 + 3 \cdot 4x = 2x^2 + 12x$
Dus: $2x^2 + 12x = 16$
$x^2 + 6x - 8 = 0$
$(x + 3)^2 = 17$
$x = -3 + \sqrt{17}$ of $x = -3 - \sqrt{17}$
Maar $x > 0$, dus alleen $x = -3 + \sqrt{17}$ voldoet.

b $2(2x^2 + 12x) = 16$
$2x^2 + 12x = 8$
$x^2 + 6x = 4$
$(x + 3)^2 - 9 = 4$
$(x + 3)^2 = 13$
$x = -3 + \sqrt{13}$ of $x = -3 - \sqrt{13}$
Maar $x > 0$, dus alleen $x = -3 + \sqrt{13}$ voldoet.

SUPER OPGAVEN

6 a -

b $x + 4 \rightarrow x^2 + 8x + 16 \rightarrow x^2 + 8x + 15$
$\rightarrow (x + 3)(x + 5)$
$y + 10 \rightarrow y^2 + 20y + 100 \rightarrow y^2 + 20y + 99$
$\rightarrow (y + 9)(y + 11)$
$z + 11 \rightarrow z^2 + 22y + 121 \rightarrow z^2 + 22z + 120$
$\rightarrow (z + 10)(z + 12)$
$p + 1 \rightarrow p^2 + 2p + 1 \rightarrow p^2 + 2p$
$\rightarrow p(p + 2)$

c -

d $(x + a)^2 = (x + a - 1)(x + a + 1)$

7 a $x^2 - 4x - 21 = 0$
$(x + 3)(x - 7) = 0$
$x = -3$ of $x = 7$

b $x^2 - 4x - 12 = 9$
$x^2 - 4x - 21 = 0$
Dus ook nu geldt: $x = -3$ of $x = 7$.

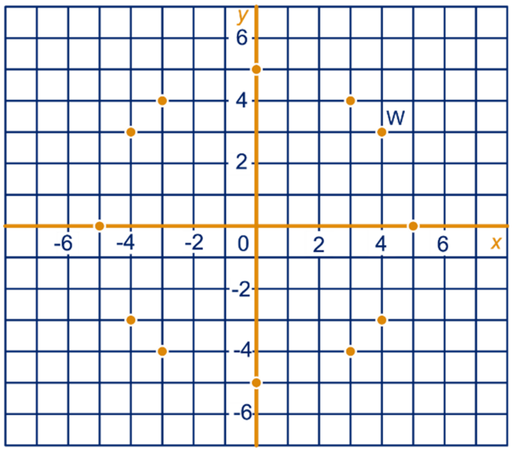
c $x^3 - 4x^2 - 21x = 0$
$x(x^2 - 4x - 21) = 0$
$x(x + 3)(x - 7) = 0$
$x = 0$ of $x = -3$ of $x = 7$

12 Als het getal p een oplossing is van $ax^2 + bx + c = 0$, dan is het getal $\frac{1}{p}$ een oplossing van $cx^2 + bx + a = 0$.

Voorbeeld
Het getal 3 is een oplossing van de vergelijking $x^2 - 4x + 3 = 0$.
Door invullen kun je nagaan dat het getal $\frac{1}{3}$ een oplossing is van $3x^2 - 4x + 1 = 0$.

Bewijs
Stel het getal p is een oplossing van $ax^2 + bx + c = 0$. Dus $ap^2 + bp + c = 0$.
We vullen het getal $\frac{1}{p}$ in de uitdrukking $cx^2 + bx + a$ in. We krijgen:
$c\cdot\left(\frac{1}{p}\right)^2 + b\cdot\frac{1}{p} + a = \left(\frac{1}{p}\right)^2 (c + bp + ap^2)$
Omdat $ap^2 + bp + c = 0$ geldt:
$c\cdot\left(\frac{1}{p}\right)^2 + b\cdot\frac{1}{p} + a = \left(\frac{1}{p}\right)^2 \cdot 0 = 0.$
Dus $\frac{1}{p}$ is een oplossing van de vergelijking $cx^2 + bx + a = 0$.

32 abcf



d $WA = \sqrt{9^2 + 3^2} = \sqrt{90} = 3\sqrt{10}$ en
$WB = \sqrt{3^2 + 1^2} = \sqrt{10}$
e Geldt $WA^2 + WB^2 = AB^2$?
Ja, want $\sqrt{90}^2 + \sqrt{10}^2 = 90 + 10 = 100 = 10^2$,
dus hoek W is recht.
g $\sqrt{(x-5)^2 + y^2}$
h $PA^2 + PB^2 = AB^2$, dus
$(x + 5)^2 + y^2 + (x - 5)^2 + y^2 = 100$
i $x^2 + 10x + 25 + y^2 + x^2 - 10x + 25 + y^2 = 100$
$2x^2 + 2y^2 + 50 = 100$
$x^2 + y^2 = 25$
Middelpunt (0,0) en straal 5.

28.8 EXTRA OPGAVEN

1 a $x(x - 5) = 2\cdot 7$
$x^2 - 5x = 14$
$x^2 - 5x - 14 = 0$
$(x - 7)(x + 2) = 0$
$x = 7$ of $x = -2$
Beide oplossingen voldoen.

b $2x^2 - 4x - 7 = 0$
$x^2 - 2x - 3\frac{1}{2} = 0$
$(x - 1)^2 - 1 - 3\frac{1}{2} = 0$
$(x - 1)^2 = 4\frac{1}{2} = \frac{18}{4}$
$x - 1 = \sqrt{\frac{18}{4}} = \frac{1}{2}\sqrt{18} = 1\frac{1}{2}\sqrt{2}$ of $x - 1 = -1\frac{1}{2}\sqrt{2}$
$x = 1 + 1\frac{1}{2}\sqrt{2}$ of $x = 1 - 1\frac{1}{2}\sqrt{2}$

c $x^2 - 5x - 5 = 0$
$(x - 2\frac{1}{2})^2 - 6\frac{1}{4} - 5 = 0$
$(x - 2\frac{1}{2})^2 = 11\frac{1}{4} = \frac{45}{4}$
$x - 2\frac{1}{2} = \sqrt{\frac{45}{4}} = \frac{1}{2}\sqrt{45} = 1\frac{1}{2}\sqrt{5}$ of $x - 2\frac{1}{2} = -1\frac{1}{2}\sqrt{5}$
$x = 2\frac{1}{2} + 1\frac{1}{2}\sqrt{5}$ of $x = 2\frac{1}{2} - 1\frac{1}{2}\sqrt{5}$

d $4(x + 1)^2 = 25\cdot 1$
$(x + 1)^2 = \frac{25}{4}$
$x + 1 = \sqrt{\frac{25}{4}} = \frac{5}{2} = 2\frac{1}{2}$ of $x + 1 = -2\frac{1}{2}$
$x = 1\frac{1}{2}$ of $x = -3\frac{1}{2}$
Beide oplossingen voldoen.

e $2x^2 + 8x + 10 = 4x^2$
$2x^2 - 8x - 10 = 0$
$x^2 - 4x - 5 = 0$
$(x - 5)(x + 1) = 0$
$x = 5$ of $x = -1$

f $x^2 - 2x + 4 = 0$
$(x - 1)^2 = -3$
Er zijn geen oplossingen.

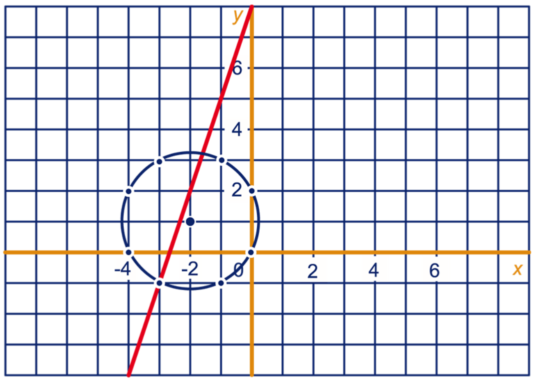
g $x^2 = x + 2$
$x^2 - x - 2 = 0$
$(x - 2)(x + 1) = 0$
$x = 2$ of $x = -1$, maar $x = -1$ mag niet.

h $(x + \frac{1}{2})^2 - \frac{1}{4} = 35\frac{3}{4}$
$(x + \frac{1}{2})^2 = 36$
$x + \frac{1}{2} = 6$ of $x + \frac{1}{2} = -6$
$x = 5\frac{1}{2}$ of $x = -6\frac{1}{2}$

i $x + 1 = 49$
$x = 48$

j $2\cdot x^2 = 1\cdot(x^2 + 3x)$
$x^2 - 3x = 0$
$x(x - 3) = 0$
$x = 0$ of $x = 3$
$x = 0$ maakt de noemers 0, dus de enige oplossing is $x = 3$.

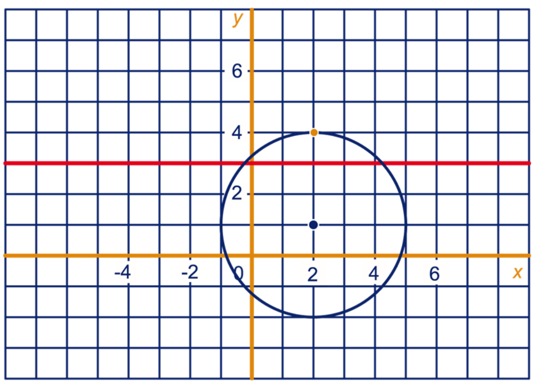
2 ab



c Vergelijking k:
$rc_k = \frac{2--1}{-2--3} = \frac{3}{1} = 3$
$b = 2 + 2\cdot 3 = 8$
dus k: $y = 3x + 8$

Vergelijking cirkel: $(x + 2)^2 + (y - 1)^2 = \sqrt{5}^2 = 5$
Snijpunten bepalen:
$(x + 2)^2 + (3x + 8 - 1)^2 = 5$
$10x^2 + 46x + 53 = 5$
$x^2 + 4{,}6x + 4{,}8 = 0$
$(x + 2{,}3)^2 = 5{,}29 - 4{,}8 = 0{,}49$
$x = \sqrt{0{,}49} - 2{,}3 = -1{,}6$ of $x = -\sqrt{0{,}49} - 2{,}3 = -3$
Als $x = -1{,}6$, dan $y = 3\cdot -1{,}6 + 8 = 3{,}2$.
Als $x = -3$, dan $y = 3\cdot -3 + 8 = -1$.
Snijpunten (-1,6 ; 3,2) en (-3,-1).

3 a



b C: $(x - 2)^2 + (y - 1)^2 = 9$
Snijpunt bepalen:
$(x - 2)^2 + (3 - 1)^2 = 9$
$(x - 2)^2 = 5$
$x - 2 = \sqrt{5}$ of $x - 2 = -\sqrt{5}$
$x = 2 + \sqrt{5}$ of $x = 2 - \sqrt{5}$
$AB = 2 + \sqrt{5} - (2 - \sqrt{5}) = 2\sqrt{5}$

4 $6x + (9\frac{1}{3} - x)x = 40$
$6x + 9\frac{1}{3}x - x^2 = 40$
$x^2 - 15\frac{1}{3}x + 40 = 0$
$(x - \frac{23}{3})^2 = \frac{169}{9}$
$x = \frac{23}{3} + \frac{13}{3} = 12$ of $x = \frac{23}{3} - \frac{13}{3} = 3\frac{1}{3}$
Maar $0 < x < 6$, dus $x = 3\frac{1}{3}$ is de enige oplossing.