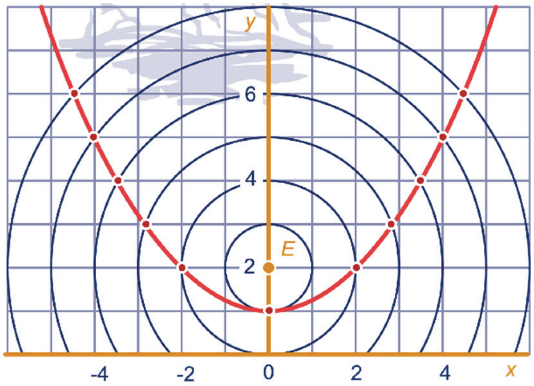
H29 PARABOLEN & HYPERBOLEN VWO

29.0 INTRO

1 ab



29.1 CONFLICTLIJN

2 a 5 ; $\sqrt{3^2+4^2}=5$

b y ; $\sqrt{x^2+(y-2)^2}$

c $y^2 = x^2+(y-2)^2$

$y^2 = x^2+y^2-4y+4$

$0 = x^2-4y+4$

$4y = x^2+4$

$y = \frac{1}{4}x^2+1$

d $3 = \frac{1}{4}x^2+1$

$2 = \frac{1}{4}x^2$

$8 = x^2$

$x = \sqrt{8} \approx 2{,}83$ of $x = -\sqrt{8} \approx -2{,}83$

$4 = \frac{1}{4}x^2+1$

$3 = \frac{1}{4}x^2$

$12 = x^2$

$x = \sqrt{12} \approx 3{,}46$ of $x = -\sqrt{12} \approx -3{,}46$

$6 = \frac{1}{4}x^2+1$

$5 = \frac{1}{4}x^2$

$20 = x^2$

$x = \sqrt{20} \approx 4{,}47$ of $x = -\sqrt{20} \approx -4{,}47$

3 a *P*: Afstand tot *E* is

$\sqrt{(1\frac{1}{2})^2+(1\frac{3}{4})^2} = \sqrt{\frac{85}{16}} = \frac{1}{4}\sqrt{85}$.

Afstand tot *k* is $2\frac{1}{4}$.

Q: Afstand tot *E* is

$\sqrt{(-3)^2+(8\frac{3}{4})^2} = \sqrt{\frac{1369}{16}} = \frac{37}{4} = 9\frac{1}{4}$.

Afstand tot *k* is $9\frac{1}{4}$.

R: Afstand tot *E* is

$\sqrt{8^2+(63\frac{3}{4})^2} = \sqrt{\frac{66.049}{16}} = \frac{257}{4} = 64\frac{1}{4}$.

Afstand tot *k* is $64\frac{1}{4}$.

Dus *Q* en *R* liggen even ver van *E* als van *k*.

b De afstand tot *k* is $y+\frac{1}{4}$.

De afstand tot *E* is $\sqrt{x^2+(y-\frac{1}{4})^2}$.

Dus:

$y+\frac{1}{4} = \sqrt{x^2+(y-\frac{1}{4})^2} \Rightarrow$

$(y+\frac{1}{4})^2 = x^2+(y-\frac{1}{4})^2$

c $(y+\frac{1}{4})^2 = x^2+(y-\frac{1}{4})^2$

$y^2+\frac{1}{2}y+\frac{1}{16} = x^2+y^2-\frac{1}{2}y+\frac{1}{16}$

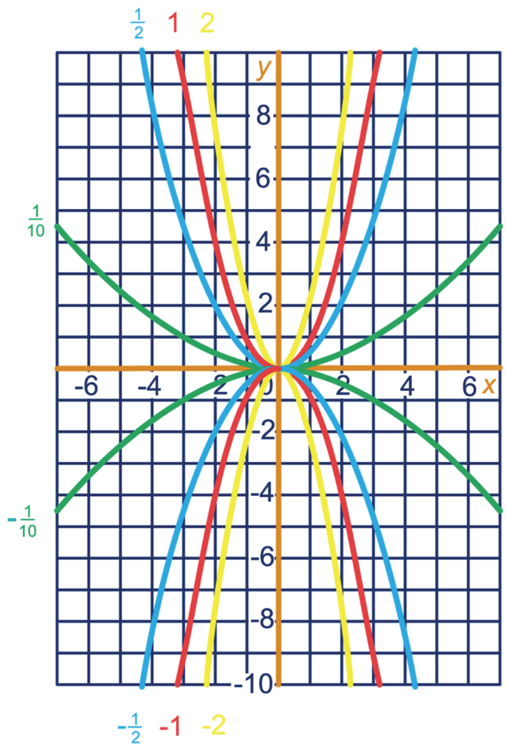
$y = x^2$

29.2 PARABOLEN

4 a

x	-3	-2	-1	0	1	2	3
$y=x^2$	9	4	1	0	1	4	9
$y=\frac{1}{10}x^2$	0,9	0,4	0,1	0	0,1	0,4	0,9
$y=\frac{1}{2}x^2$	4,5	2	0,5	0	0,5	2	4,5
$y=2x^2$	18	8	2	0	2	8	18

bd



c

x	-3	-2	-1	0	1	2	3
$y = -x^2$	-9	-4	-1	0	-1	-4	-9
$y = -\frac{1}{10}x^2$	-0,9	-0,4	-0,1	0	-0,1	-0,4	-0,9
$y = -\frac{1}{2}x^2$	-4,5	-2	-0,5	0	-0,5	-2	-4,5
$y = -2x^2$	-18	-8	-2	0	-2	-8	-18

e Dalparabool als $c > 0$,
een bergparabool als $c < 0$.

f Ze zijn elkaars spiegelbeeld in de x-as.

g Dan is $y = 0$, dat is een rechte lijn, dat is de vergelijking van de x-as.

5 $y = cx^2$
$3 = c \cdot 1^2$ (invullen het punt (1,3))
$3 = c$

$y = cx^2$
$2 = c \cdot (-5)^2$ (invullen het punt (-5,2))
$2 = 25c$
$\frac{2}{25} = c$

$y = cx^2$
$-3 = c \cdot 3^2$ (invullen het punt (3,-3))
$-3 = 9c$
$-\frac{1}{3} = c$

6 $y = cx^2$
$4 = c \cdot 5^2$ (invullen het punt (5,4) of (-5,4))
$4 = 25c$
$\frac{4}{25} = c$

7 a $h = cx^2$
$6{,}25 = c \cdot 10^2$ (invullen het punt (10 ; 6,25))
$6{,}25 = 100c$
$\frac{1}{16} = c$
Dus $h = \frac{1}{16}x^2$

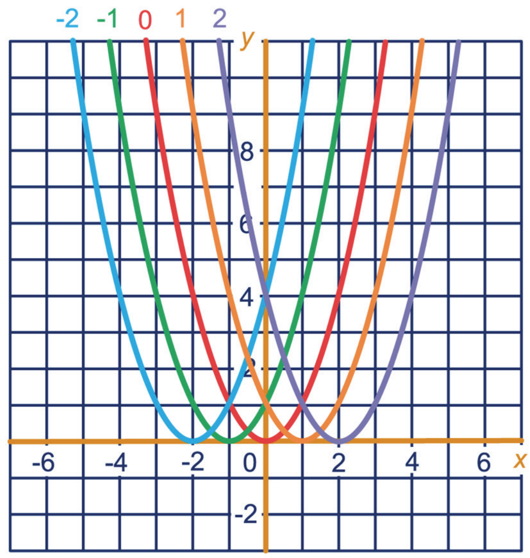
b $h = \frac{1}{16} \cdot 40^2 = 100$ m

c als $x = 0$, dan $h = \frac{1}{16} \cdot 0^2 = 0$ m
als $x = 10$, dan $h = \frac{1}{16} \cdot 10^2 = 6{,}25$ m
als $x = 20$, dan $h = \frac{1}{16} \cdot 20^2 = 25$ m
als $x = 30$, dan $h = \frac{1}{16} \cdot 30^2 = 56{,}25$ m
als $x = 40$, dan $h = \frac{1}{16} \cdot 40^2 = 100$ m

d $x = 35$, dan $h = \frac{1}{16} \cdot 35^2 = 76{,}5625$ m
De hoogte boven de Wupper is dan
$100 - 76{,}5625 = 23{,}4375$ m

8 a

x	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = (x-1)^2$	16	9	4	1	0	1	4
$y = (x-2)^2$	25	16	9	4	1	0	1
$y = (x+1)^2$	4	1	0	1	4	9	16
$y = (x+2)^2$	1	0	1	4	9	16	25

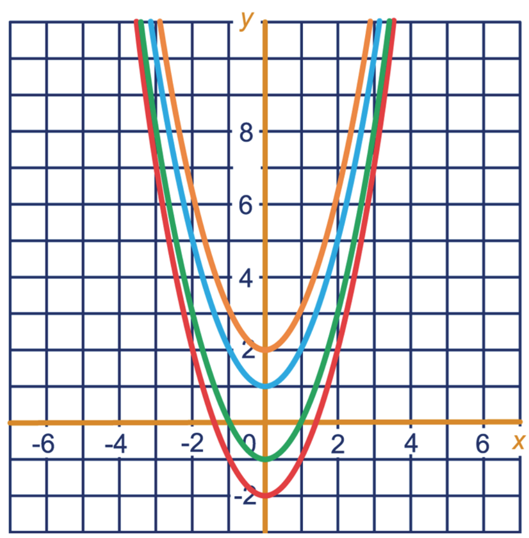


b Eén eenheid naar rechts.

c Door de grafiek twee eenheden naar links te schuiven.

9 a

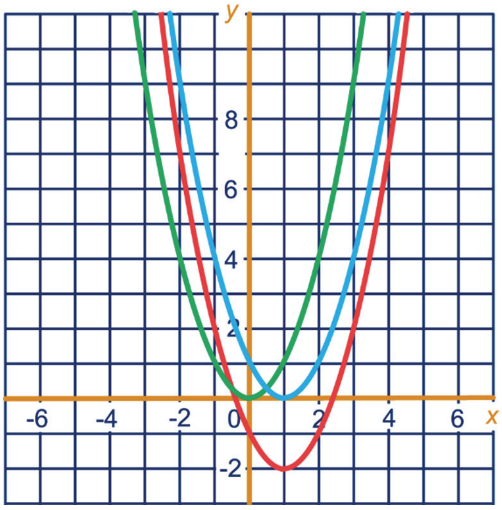
x	-3	-2	-1	0	1	2	3
$y = x^2 + 1$	10	5	2	1	2	5	10
$y = x^2 + 2$	11	6	3	2	3	6	11
$y = x^2 - 1$	8	3	0	-1	0	3	8
$y = x^2 - 2$	7	2	-1	-2	-1	2	7



b

x	-3	-2	-1	0	1	2	3
$y = (x-1)^2$	16	9	4	1	0	1	4
$y = (x-1)^2 - 2$	14	7	2	-1	-2	-1	2

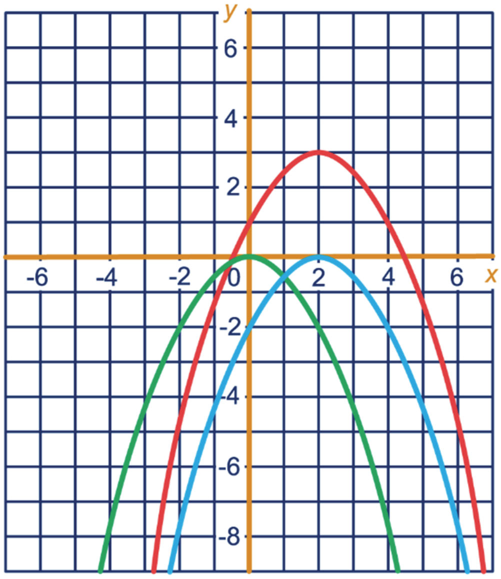
c



10 a

x	-2	-1	0	1	2	3	4
$y=-\frac{1}{2}x^2$	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2	$-4\frac{1}{2}$	-8
$y=-\frac{1}{2}(x-2)^2$	-8	$-4\frac{1}{2}$	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2
$y=-\frac{1}{2}(x-2)^2+3$	-5	$-1\frac{1}{2}$	1	$2\frac{1}{2}$	3	$2\frac{1}{2}$	1

b



c 2 ; rechts
3 ; boven

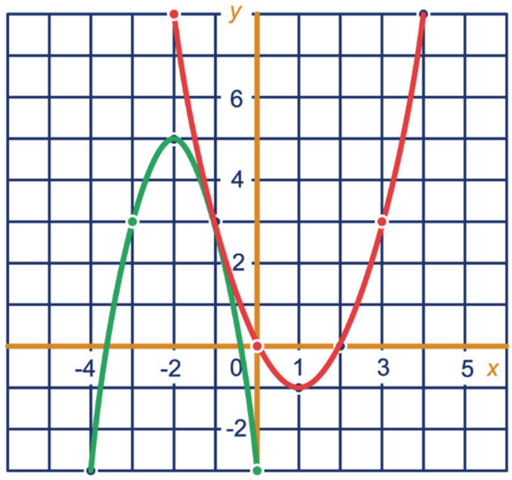
11 a 3 eenheden naar links
b 4 eenheden naar beneden

12 (2,3) ; (-3,-4)

13 (2,2) ; (-3,0)
(3,2) ; (0,3)

14 a (-1,2)
b $y = (2 + 1)^2 + 2 = 11$. Gaat niet door (2,20).
$y = -(2 + 1)^2 + 2 = -7$. Gaat niet door (2,20).
$y = 2(2 + 1)^2 + 2 = 20$. Gaat door (2,20).

15



16 Het punt (10,-3) ligt 10 – 4 = 6 eenheden rechts van de symmetrieas, dus het ander punt ligt 6 eenheden links van de symmetrieas op (-2,-3).
Het punt (-1,19) ligt 4 – -1 = 5 eenheden links van de symmetrieas, dus het ander punt ligt 5 eenheden rechts van de symmetrieas op (9,19).

29.3 VERGELIJKINGEN OPSTELLEN VOOR PARABOLEN

17 a $y = cx^2$
$3 = c \cdot 4^2$ (invullen het punt (4,3))
$3 = 16c$
$\frac{3}{16} = c$
Vergelijking parabool: $y = \frac{3}{16}x^2$.

b $x = 3$ of $x = -3 \Rightarrow y = \frac{3}{16} \cdot 3^2 = 1\frac{11}{16}$
Dus $(3, 1\frac{11}{16})$ en $(-3, 1\frac{11}{16})$.

18 a $y = cx^2$
$62{,}5 = c \cdot 250^2$ (invullen het punt (250 ; 62,5))
$62{,}5 = 62.500c$
$\frac{1}{1000} = c$

b $a = 500$ en $b = 0$

c $y = c(x - 500)^2 + 0$
$62{,}5 = c(250 - 500)^2$ (invullen (250 ; 62,5))
$62{,}5 = 62.500c$
$\frac{1}{1000} = c$

19 a $y = -2(x - 3)^2 + 2$:
bergparabool met top (3,2), dus D.
$y = -\frac{1}{2}(x + 3)^2 + 2$:
bergparabool met top (-3,2), dus A.

b Top_B (-2,-4)
$y = c(x + 2)^2 - 4$
$6 = c(1 + 2)^2 - 4$ (invullen (1,6))
$6 = 9c - 4$
$10 = 9c$
$1\frac{1}{9} = c$
Vergelijking B: $y = 1\frac{1}{9}(x + 2)^2 - 4$.

Top$_C$ (3,2)
$y = c(x-3)^2 + 2$
$3 = c(5-3)^2 + 2$ (invullen (5,3))
$3 = 4c + 2$
$1 = 4c$
$\frac{1}{4} = c$
Vergelijking C: $y = \frac{1}{4}(x-3)^2 + 2$.

Top$_E$ (5,-2)
$y = c(x-5)^2 - 2$
$-4 = c(6-5)^2 - 2$ (invullen (6,-4))
$-4 = c - 2$
$-2 = c$
Vergelijking E: $y = -2(x-5)^2 - 2$.

29.4 *abc*-FORMULE

20 a $y = (x+1)^2 + 3 = x^2 + 2x + 4$

b $y = x^2 + 4x - 3 = (x+2)^2 - 7$

c $y = 2x^2 + 8x - 6$
$y = 2(x^2 + 4x - 3)$
$y = 2((x+2)^2 - 7)$
$y = 2(x+2)^2 - 14$

d (-2,-14)

e $y = \frac{1}{2}x^2 + 3x + 2$
$y = \frac{1}{2}(x^2 + 6x + 4)$
$y = \frac{1}{2}((x+3)^2 - 5)$
$y = \frac{1}{2}(x+3)^2 - 2\frac{1}{2}$

f $(-3,-2\frac{1}{2})$

g $y = -x^2 + x$
$y = -(x^2 - x)$
$y = -((x - \frac{1}{2})^2 - \frac{1}{4})$
$y = -(x - \frac{1}{2})^2 + \frac{1}{4}$, Top $(\frac{1}{2}, \frac{1}{4})$

$y = -2x^2 + 10x + 1$
$y = -2(x^2 - 5x - \frac{1}{2})$
$y = -2((x - 2\frac{1}{2})^2 - 6\frac{3}{4})$
$y = -2(x - 2\frac{1}{2})^2 + 13\frac{1}{2}$, Top $(2\frac{1}{2}, 13\frac{1}{2})$

$y = -(2x+4)^2 + 4$, Top (-2,4)

$y = (x+3)(x-7)$, snijpunten met de x-as zijn: (-3,0) en (7,0). De symmetrieas ligt daar midden tussen.
Symmetrieas van de parabool: $x = \frac{-3+7}{2} = 2$,
$y = (2+3)(2-7) = -25$, Top (2,-25).

21 $x = -\frac{11}{6} + \sqrt{\frac{25}{36}} = -1$ of $x = -\frac{11}{6} - \sqrt{\frac{25}{36}} = -2\frac{2}{3}$

22 a $x = -\frac{9}{2\cdot5} + \sqrt{(\frac{9}{2\cdot5})^2 - \frac{2}{5}}$ of $x = -\frac{9}{2\cdot5} - \sqrt{(\frac{9}{2\cdot5})^2 - \frac{2}{5}}$

b $x = -\frac{9}{2\cdot5} + \sqrt{(\frac{9}{2\cdot5})^2 - \frac{2}{5}} = \frac{9}{10} + \sqrt{\frac{41}{100}} = -\frac{9}{10} + \frac{1}{10}\sqrt{41}$
of
$x = -\frac{9}{2\cdot5} - \sqrt{(\frac{9}{2\cdot5})^2 - \frac{2}{5}} = -\frac{9}{10} - \frac{1}{10}\sqrt{41}$

23 $x = -\frac{b}{2a} + \sqrt{(\frac{b}{2a})^2 - \frac{c}{a}}$ of $x = -\frac{b}{2a} - \sqrt{(\frac{b}{2a})^2 - \frac{c}{a}}$

24 a $x = -\frac{-5}{2\cdot2} + \sqrt{(\frac{-5}{2\cdot2})^2 - \frac{-25}{2}} = \frac{5}{4} + \sqrt{\frac{225}{16}} = \frac{5}{4} + \frac{15}{4} = \frac{20}{4} = 5$
of
$x = -\frac{-5}{2\cdot2} - \sqrt{(\frac{-5}{2\cdot2})^2 - \frac{-25}{2}} = \frac{5}{4} - \frac{15}{4} = -\frac{10}{4} = -2\frac{1}{2}$

b $2\cdot5^2 - 5\cdot5 - 25 = 50 - 25 - 25 = 0$
$2\cdot(-2\frac{1}{2})^2 - 5\cdot -2\frac{1}{2} - 25 = 12\frac{1}{2} + 12\frac{1}{2} - 25 = 0$

25 a $x^2 + 4x + 5 = 0$
$(x+2)^2 - 4 + 5 = 0$
$(x+2)^2 = -1$ en dat kan voor geen enkele x.

b $\sqrt{(\frac{4}{2\cdot1})^2 - \frac{5}{1}} = \sqrt{4-5} = \sqrt{-1}$, maar $\sqrt{-1}$ bestaat niet.

c stap 1:
haakjes uitwerken, $(\frac{b}{2a})^2 = \frac{b}{2a}\cdot\frac{b}{2a} = \frac{b^2}{4a^2}$
stap 2:
breuken gelijknamig maken, $\frac{c}{a}\cdot\frac{4a}{4a} = \frac{4ac}{4a^2}$
stap 3:
twee breuken met dezelfde noemer optellen:
de noemer zo laten en de tellers optellen.

d als $a > 0$, dan $2a\cdot2a = 4a^2$ en
als $a < 0$, dan $-2a\cdot-2a = 4a^2$

e als $a > 0$, $-\frac{b}{2a} + \sqrt{\frac{D}{4a^2}} = -\frac{b}{2a} + \frac{\sqrt{D}}{2a}$
of: $-\frac{b}{2a} - \sqrt{\frac{D}{4a^2}} = -\frac{b}{2a} - \frac{\sqrt{D}}{2a}$

als $a < 0$, $-\frac{b}{2a} + \sqrt{\frac{D}{4a^2}} = -\frac{b}{2a} + \frac{\sqrt{D}}{-2a} = -\frac{b}{2a} - \frac{\sqrt{D}}{2a}$
of: $-\frac{b}{2a} - \sqrt{\frac{D}{4a^2}} = -\frac{b}{2a} - \frac{\sqrt{D}}{-2a} = -\frac{b}{2a} + \frac{\sqrt{D}}{2a}$

f $D = 2^2 - 4\cdot1\cdot3 = -8$
$D = 3^2 - 4\cdot-1\cdot2 = 17$
$D = 20^2 - 4\cdot4\cdot25 = 0$

g geen oplossingen
$x = \frac{-3+\sqrt{17}}{2\cdot-1} = 1\frac{1}{2} - \frac{1}{2}\sqrt{17}$ of $x = 1\frac{1}{2} + \frac{1}{2}\sqrt{17}$
$x = -\frac{20}{8} = -2\frac{1}{2}$

26 Dan staat er een lineaire vergelijking.

27 $2x^2 - 3x - 35 = 0$
$\left.\begin{array}{l} a = 2 \\ b = -3 \\ c = -35 \end{array}\right\} D = 9 - 4\cdot2\cdot-35 = 289$, $\sqrt{D} = 17$
$x = \frac{3+17}{4} = 5$ of $x = \frac{3-17}{4} = -3\frac{1}{2}$

$2x^2+4x-1=0$

$\left.\begin{array}{l}a=2\\b=4\\c=-1\end{array}\right\}\begin{array}{l}D=16-4\cdot2\cdot-1=24\\\sqrt{D}=\sqrt{24}=2\sqrt{6}\end{array}$

$x=\frac{-4+2\sqrt{6}}{4}=-1+\frac{1}{2}\sqrt{6}$ of $x=\frac{-4-2\sqrt{6}}{4}=-1-\frac{1}{2}\sqrt{6}$

$7x^2-6x+2=0$

$\left.\begin{array}{l}a=7\\b=-6\\c=2\end{array}\right\}D=36-4\cdot7\cdot2=-20$

$D<0$, dus géén oplossingen

$\frac{1}{2}x^2-3x-4\frac{1}{2}=0$

$\left.\begin{array}{l}a=\frac{1}{2}\\b=-3\\c=-4\frac{1}{2}\end{array}\right\}\begin{array}{l}D=9-4\cdot\frac{1}{2}\cdot-4\frac{1}{2}=18\\\sqrt{D}=\sqrt{18}=3\sqrt{2}\end{array}$

$x=\frac{3+3\sqrt{2}}{1}=3+3\sqrt{2}$ of $x=\frac{3-3\sqrt{2}}{1}=3-3\sqrt{2}$

$4x=1+4x^2$

$4x^2-4x+1=0$

$\left.\begin{array}{l}a=4\\b=-4\\c=1\end{array}\right\}D=16-4\cdot4\cdot1=0$

$x=-\frac{-4}{8}=\frac{1}{2}$

$(x-3)^2=5-3x$

$x^2-6x+9=5-3x$

$x^2-3x+4=0$

$\left.\begin{array}{l}a=1\\b=-3\\c=4\end{array}\right\}D=9-4\cdot1\cdot4=-7$

$D<0$, dus geen oplossingen

$5x-3x^2=0$

$\left.\begin{array}{l}a=-3\\b=5\\c=0\end{array}\right\}\begin{array}{l}D=25-4\cdot-3\cdot0=25\\\sqrt{D}=5\end{array}$

$x=\frac{-5+5}{-6}=0$ of $x=\frac{-5-5}{-6}=\frac{10}{6}=1\frac{2}{3}$

28 $(x+3)^2=16$

$x+3=4$ of $x+3=-4$

$x=1$ of $x=-7$

$(x+1)^2=(2x+3)^2$

$x+1=2x+3$ of $x+1=-2x-3$

$-2=x$ of $3x=-4$

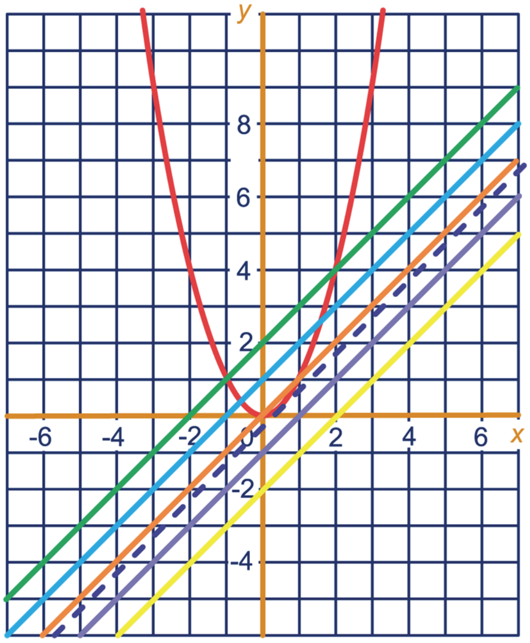
$x=-\frac{4}{3}=-1\frac{1}{3}$

$x^2+6x+5=0$

$(x+1)(x+5)=0$

$x=-1$ of $x=-5$

29 ab



c $x^2=x$

$x^2-x=0$

$x(x-1)=0$

$x=0$ of $x=1$

Snijpunten: (0,0) en (1,1)

d $x^2=x+2$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2$ of $x=-1$

Snijpunten: (2,4) en (-1,1)

e $x^2=x-2$

$x^2-x+2=0$

$\left.\begin{array}{l}a=1\\b=-1\\c=2\end{array}\right\}D=1-4\cdot1\cdot2=-7$

$D<0$, dus geen snijpunten.

f Alle lijnen hebben richtingscoëfficiënt 1.

g Zie blauwe stippellijn in opgave **a**.

h $x^2 = x - 1$

$x^2 - x + 1 = 0$

$\left.\begin{array}{l} a = 1 \\ b = -1 \\ c = 1 \end{array}\right\} D = 1 - 4 \cdot 1 \cdot 1 = -3$

$x^2 = x$

$x^2 - x = 0$

$\left.\begin{array}{l} a = 1 \\ b = -1 \\ c = 0 \end{array}\right\} D = 1 - 4 \cdot 1 \cdot 0 = 1$

$x^2 = x + 1$

$x^2 - x - 1 = 0$

$\left.\begin{array}{l} a = 1 \\ b = -1 \\ c = -1 \end{array}\right\} D = 1 - 4 \cdot 1 \cdot -1 = 5$

i $x^2 - x - k = 0$

$\left.\begin{array}{l} a = 1 \\ b = -1 \\ c = -k \end{array}\right\} D = 1 - 4 \cdot 1 \cdot -k = 1 + 4k$

j $1 + 4k = 0$

$4k = -1$

$k = -\frac{1}{4}$

k $x^2 = x - \frac{1}{4}$

$x^2 - x + \frac{1}{4} = 0$

$x = -\frac{-1}{2} = \frac{1}{2} \Rightarrow y = (\frac{1}{2})^2 = \frac{1}{4}$

Raakpunt is $(\frac{1}{2}, \frac{1}{4})$.

30 a $-x^2 + 1 = ax + 3$

$x^2 + ax + 2 = 0$

b $x^2 + ax + 2 = 0$

$\left.\begin{array}{l} a = 1 \\ b = a \\ c = 2 \end{array}\right\} D = a^2 - 4 \cdot 1 \cdot 2 = a^2 - 8$

c Raken $\Rightarrow D = 0$, dus

$a^2 - 8 = 0$

$a^2 = 8$

$a = \sqrt{8} = 2\sqrt{2}$ of $a = -\sqrt{8} = -2\sqrt{2}$

d $x^2 + 2\sqrt{2}x + 2 = 0$

$x = -\frac{2\sqrt{2}}{2} = -\sqrt{2} \Rightarrow y = -(-\sqrt{2})^2 + 1 = -1$

Raakpunt is $(-\sqrt{2},-1)$.

$x^2 - 2\sqrt{2}x + 2 = 0$

$x = -\frac{-2\sqrt{2}}{2} = \sqrt{2} \Rightarrow y = -(\sqrt{2})^2 + 1 = -1$

Raakpunt is $(\sqrt{2},-1)$.

31 a $x^2 + 5x + 4 = 0$

$(x+1)(x+4) = 0$

$x = -1$ of $x = -4$

b $x^2 + kx + 4 = 0$

$\left.\begin{array}{l} a = 1 \\ b = k \\ c = 4 \end{array}\right\} D = k^2 - 4 \cdot 1 \cdot 4 = k^2 - 16$

Eén oplossing, dus $D = 0$.

$k^2 - 16 = 0$

$k^2 = 16$

$k = 4$ of $k = -4$

c $x^2 + 4x + 4 = 0$

$(x+2)^2 = 0$

$x = -2$

32 a x-as $\Rightarrow y = 0$, dan:

$px^2 - 6x - 1 = 0$

$\left.\begin{array}{l} a = p \\ b = -6 \\ c = -1 \end{array}\right\} D = 36 - 4 \cdot p \cdot -1 = 36 + 4p$

Eén raakpunt met x-as $\Rightarrow D = 0$

$36 + 4p = 0$

$4p = -36$

$p = -9$

b x-as $\Rightarrow y = 0$, dan:

$\frac{1}{2}x^2 - px + 2 = 0$

$\left.\begin{array}{l} a = \frac{1}{2} \\ b = -p \\ c = 2 \end{array}\right\} D = (-p)^2 - 4 \cdot \frac{1}{2} \cdot 2 = p^2 - 4$

Twee snijpunten met x-as $\Rightarrow D > 0$

$p^2 - 4 > 0$

$p^2 > 4$

$p < -2$ of $p > 2$

c x-as $\Rightarrow y = 0$, dan:

$2x^2 + 4x + p = 0$

$\left.\begin{array}{l} a = 2 \\ b = 4 \\ c = p \end{array}\right\} D = 16 - 4 \cdot 2 \cdot p = 16 - 8p$

Geen snij- of raakpunten met x-as $\Rightarrow D < 0$

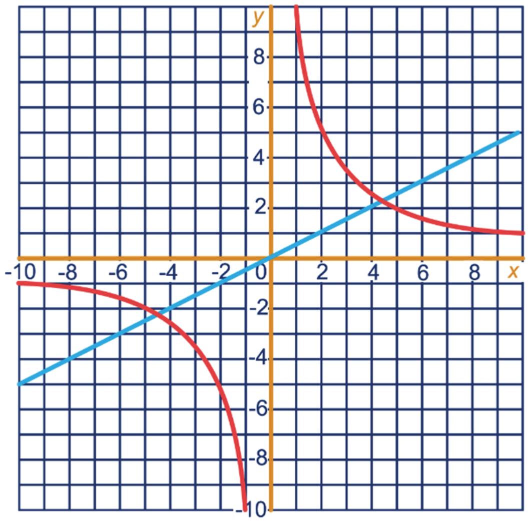
$16 - 8p < 0$

$16 < 8p$

$2 < p$

29.5 HYPERBOLEN

33 a Zie de twee rode lijnen hieronder.



b $-1\frac{1}{2}\cdot -7 = 10\frac{1}{2}$, ligt er niet op

$\sqrt{20}\cdot\sqrt{5} = \sqrt{100} = 10$, ligt er op

$-\sqrt{10}\cdot -\sqrt{10} = \sqrt{100} = 10$, ligt er op

$8\cdot 1\frac{1}{4} = 10$, ligt er op

c $10 : \frac{5}{3} = 6$

$10 : -2\frac{1}{2} = -4$

$10 : 2\sqrt{5} = \sqrt{5}$

$10 : -5\sqrt{2} = -\sqrt{2}$

d $(\sqrt{10},\sqrt{10})$ en $(-\sqrt{10},-\sqrt{10})$

e Zie blauwe lijn hierboven.

f $y = \frac{1}{2}x$

g $x\cdot\frac{1}{2}x = 10$

$\frac{1}{2}x^2 = 10$

$x^2 = 20$

$x = \sqrt{20} = 2\sqrt{5}$ of $x = -\sqrt{20} = -2\sqrt{5}$

Als $x = 2\sqrt{5}$, dan $y = \frac{1}{2}x = \sqrt{5}$.

Als $x = -2\sqrt{5}$, dan $y = -\sqrt{5}$.

Snijpunten zijn $(2\sqrt{5},\sqrt{5})$ en $(-2\sqrt{5},-\sqrt{5})$.

h $10 : 4 = 2\frac{1}{2} = 2{,}5$

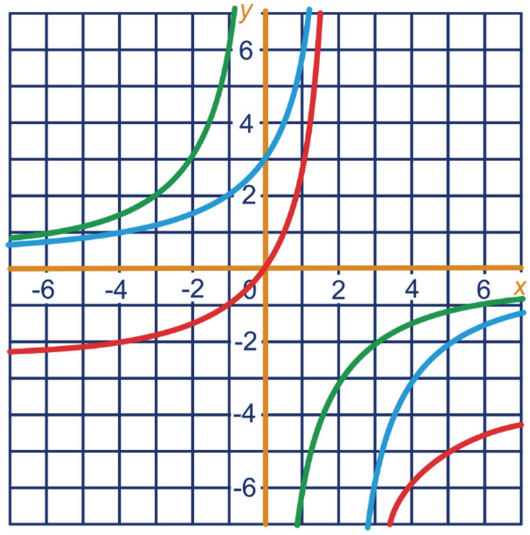
$10 : 40 = \frac{1}{4} = 0{,}25$

$10 : 400 = \frac{1}{40} = 0{,}025$

$10 : 4000 = \frac{1}{400} = 0{,}0025$

i De y-as, en de vergelijking van de y-as is $x = 0$.

34 abe



c twee eenheden naar rechts

d horizontale asymptoot: $y = 0$
verticale asymptoot: $x = 2$

f horizontale asymptoot: $y = -3$
verticale asymptoot: $x = 2$

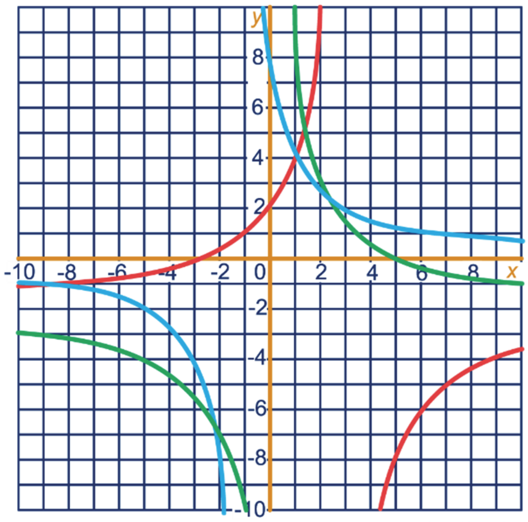
g drie eenheden naar beneden

35 a horizontale asymptoot: $y = -2$
verticale asymptoot: $x = 0$

horizontale asymptoot: $y = -2$
verticale asymptoot: $x = 3$

horizontale asymptoot: $y = 0$
verticale asymptoot: $x = -1$

b



36 a $(x + 1)(y - 3) = 12$
horizontale asymptoot: $y = 3$
verticale asymptoot: $x = -1$
Dus A.

$(x-2)(y+4) = -8$
horizontale asymptoot: $y = -4$
verticale asymptoot: $x = 2$
Dus D.

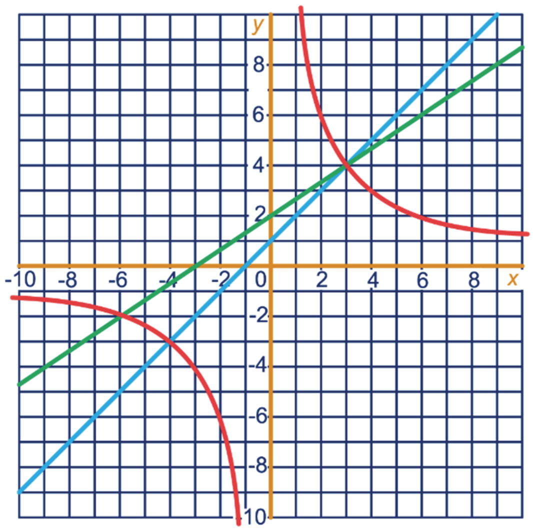
b horizontale asymptoot B: $y = 2$
verticale asymptoot B: $x = -1$
$(x+1)(y-2) = c$
$(-5+1)(6-2) = c$ (invullen (-5,6))
$-16 = c$
Vergelijking B: $(x+1)(y-2) = -16$.

horizontale asymptoot C: $y = 0$
verticale asymptoot C: $x = 0$
$xy = c$
$-4 \cdot -6 = c$ (invullen (-4,-6))
$24 = c$
Vergelijking C: $xy = 24$.

horizontale asymptoot E: $y = 2$
verticale asymptoot E: $x = 4$
$(x-4)(y-2) = c$
$(7-4)(4-2) = c$ (invullen (7,4))
$6 = c$
Vergelijking E: $(x-4)(y-2) = 6$.

29.6 GEMENGDE OPGAVEN

37 ac



b $x(x+1) = 12$

$x^2 + x - 12 = 0$

$(x-3)(x+4) = 0$

$x = 3$ of $x = -4$

Als $x = 3$, dan $y = 3 + 1 = 4$.

Als $x = -4$, dan $y = -4 + 1 = -3$.

Snijpunten zijn $(3,4)$ en $(-4,-3)$.

d $2x - 3y + 6 = 0$

$2x + 6 = 3y$

$\frac{2}{3}x + 2 = y$

Vergelijking:

$x(\frac{2}{3}x + 2) = 12$

$\frac{2}{3}x^2 + 2x - 12 = 0$

$x^2 + 3x - 18 = 0$

$(x-3)(x+6) = 0$

$x = 3$ of $x = -6$

Als $x = 3$, dan $y = \frac{12}{3} = 4$.

Als $x = -6$, dan $y = \frac{12}{-6} = -2$.

Snijpunten zijn $(3,4)$ en $(-6,-2)$.

e $x(-x + k) = 12$

$-x^2 + kx - 12 = 0$

$x^2 - kx + 12 = 0$

$$\left.\begin{array}{l} a = 1 \\ b = -k \\ c = 12 \end{array}\right\} D = (-k)^2 - 4 \cdot 1 \cdot 12 = k^2 - 48$$

$k^2 - 48 = 0$

$k^2 = 48$

$k = \sqrt{48} = 4\sqrt{3}$ of $k = -4\sqrt{3}$

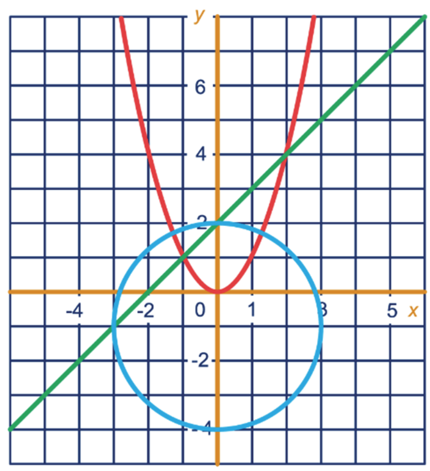
Vergelijkingen van de raaklijnen:
$y = -x + 4\sqrt{3}$ en $y = -x - 4\sqrt{3}$.

38 a $x^2 + y^2 + 2y = 8$

$x^2 + (y+1)^2 - 1 = 8$

$x^2 + (y+1)^2 = 9$

bc



d $x - y + 2 = 0 \Leftrightarrow x + 2 = y$ en $x^2 + (y+1)^2 = 9$

Vergelijking:

$x^2 + (x + 2 + 1)^2 = 9$

$x^2 + (x+3)^2 = 9$

$2x^2 + 6x = 0$

$2x(x+3) = 0$

$x = 0$ of $x = -3$

Als $x = 0$, dan $y = 0 + 2 = 2$.

Als $x = -3$, dan $y = -3 + 2 = -1$.
Snijpunten zijn $(0,2)$ en $(-3,-1)$.

e $x + 2 = y$ en $y = x^2$

$x + 2 = x^2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2$ of $x = -1$

Als $x = 2$, dan $y = 2 + 2 = 4$.
Als $x = -1$, dan $y = -1 + 2 = 1$.
Snijpunten zijn $(2,4)$ en $(-1,1)$.

f $y = x^2$ en $x^2 + (y+1)^2 = 9$

$y + (y+1)^2 = 9$

$y^2 + 3y - 8 = 0$

$\left.\begin{array}{l} a = 1 \\ b = 3 \\ c = -8 \end{array}\right\} \begin{array}{l} D = 9 - 4 \cdot 1 \cdot -8 = 41 \\ \sqrt{D} = \sqrt{41} \end{array}$

$y = \frac{-3+\sqrt{41}}{2} = -1\frac{1}{2} + \frac{1}{2}\sqrt{41}$ of $y = -1\frac{1}{2} - \frac{1}{2}\sqrt{41}$

Alleen $y = -1\frac{1}{2} + \frac{1}{2}\sqrt{41}$ voldoet, want $y \geq 0$.

39 a $y = -x^2 + 2x + 3$

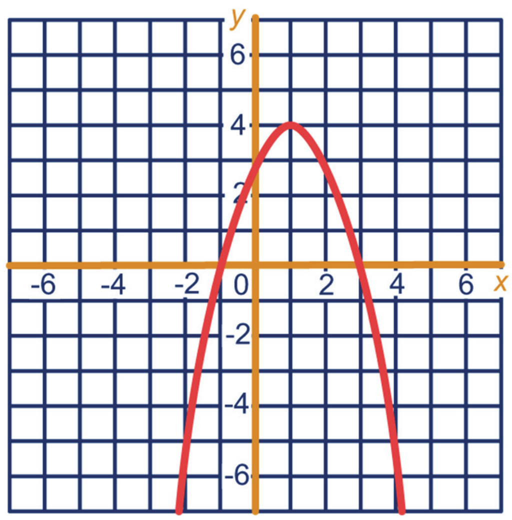
$y = -(x^2 - 2x - 3)$

$y = -((x-1)^2 - 4)$

$y = -(x-1)^2 + 4$

Top (1,4).

b



c 1 ; 4
d $y \leq 4$

40 a $-x^2 + 4x + 5 = 0$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ of $x = -1$

Snijpunten: (5,0) en (-1,0).

b Vergelijking symmetrieas: $x = \frac{5-1}{2} = 2$.
c $x = 2 \Rightarrow y = -2^2 + 4 \cdot 2 + 5 = -4 + 8 + 5 = 9$
Top (2,9).

41 a $p \cdot V = c$

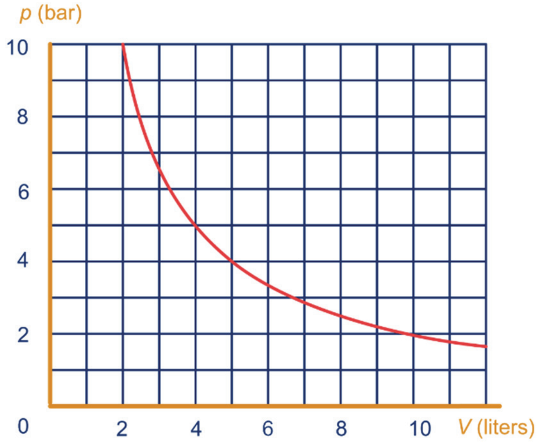
$4 \cdot 5 = c$

$20 = c$

$p \cdot 3 = 20$

$p = \frac{20}{3} = 6\frac{2}{3}$ bar

b



c $V > \frac{20}{3} = 6\frac{2}{3}$ bar

42 a $C = -20x^2 + 60x$

$C = -20(x^2 - 3x)$

$C = -20((x - 1\frac{1}{2})^2 - 2\frac{1}{4})$

$C = -20(x - 1\frac{1}{2})^2 + 45$

Top ($1\frac{1}{2}$,45).

b Een bergparabool.
c Bij hoogte $1\frac{1}{2}$, de capaciteit is dan 45.

43 parabool
cirkel
lijn
verticale lijn
hyperbool

SUPER OPGAVEN

14 y vervangen in de vergelijking
$x + y + 6 = (x - y + 3)^2$ door $x + 3$. Dus:

$x + (x+3) + 6 = (x - (x+3) + 3)^2$

$2x + 9 = 0$

$x = -4\frac{1}{2} \Rightarrow y = -4\frac{1}{2} + 3 = -1\frac{1}{2}$

Top ($-4\frac{1}{2}$, $-1\frac{1}{2}$).

16 Als de top op de y-as ligt, dan zijn (-2,4) en (3,6) ook punten van de parabool.
Dus dan moet het een dalparabool zijn.

19 a $y = c(x-4)^2 + 6$ (invullen top$(a,b) = (4,6)$)

$3 = c(1-4)^2 + 6$ (invullen punt$(x,y) = (1,3)$)

$3 = 9c + 6$

$-3 = 9c$

$-\frac{1}{3} = c$

Vergelijking parabool: $y = -\frac{1}{3}(x-4)^2 + 6$.

b Top op de y-as $\Rightarrow a = 0 \Rightarrow y = cx^2 + b$

$9 = c\cdot(-3)^2 + b \quad \Rightarrow \quad 9 = 9c + b$

$-3 = c\cdot 2^2 + b \quad \Rightarrow \quad \underline{-3 = 4c + b}$ _

$12 = 5c$

$2\frac{2}{5} = \frac{12}{5} = c$

$b = 9 - 9\cdot 2\frac{2}{5} = -12\frac{3}{5}$

Vergelijking parabool: $y = 2\frac{2}{5}x^2 - 12\frac{3}{5}$.

c Top op de x-as $\Rightarrow b = 0 \Rightarrow y = c(x-a)^2$

$y = \frac{1}{3}(x-a)^2$ invullen $\frac{1}{3}$ voor c

$3 = \frac{1}{3}(4-a)^2$ invullen het punt (4,3)

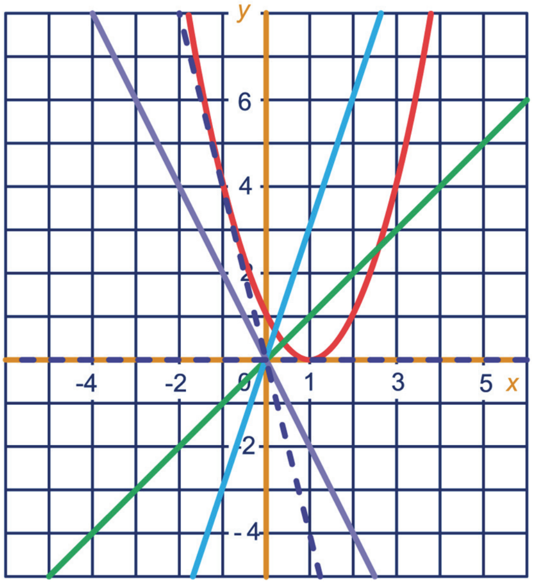
$9 = (4-a)^2$

$4 - a = 3$ of $4 - a = -3$

$a = 1$ of $a = 7$

Vergelijking symmetrieas: $x = 1$ of $x = 7$.

29 aef



b Omdat $0 = k\cdot 0$ klopt, wat je ook voor k neemt.

c Als $k = 0$, dan $y = 0$. ; $5 = k\cdot -2 \Rightarrow k = -2\frac{1}{2}$

d De verticale as, dus de y-as.

f Zie de twee stippellijnen in opgave **a**.

g $(x-1)^2 = kx$

$x^2 - 2x + 1 = kx$

$x^2 - 2x - kx + 1 = 0$

$x^2 - (2+k)x + 1 = 0$

h $D = (2+k)^2 - 4\cdot 1\cdot 1 = (2+k)^2 - 4$

i Als $k = 1$, dan $D = 9 - 4 = 5$, dus twee snijpunten.

j $(2+k)^2 - 4 = 0$

$(2+k)^2 = 4$

$2 + k = 2$ of $2 + k = -2$

$k = 0$ of $k = -4$

k $y = 0$ en $y = -4x$

31 a $x^2 + (-x+k)^2 = 3$

$2x^2 - 2kx + k^2 - 3 = 0$

$$\left.\begin{array}{l} a = 2 \\ b = -2k \\ c = k^2 - 3 \end{array}\right\} D = (-2k)^2 - 4\cdot 2\cdot(k^2-3) = 24 - 4k^2$$

Raken, dus $D = 0$.

$24 - 4k^2 = 0$

$24 = 4k^2$

$6 = k^2$

$k = \sqrt{6}$ of $k = -\sqrt{6}$

b Als $k = \sqrt{6}$:

$x = -\frac{b}{2a} = -\frac{-2\sqrt{6}}{4} = \frac{1}{2}\sqrt{6}$

$y = -\frac{1}{2}\sqrt{6} + \sqrt{6} = \frac{1}{2}\sqrt{6}$

Rechter raakpunt is $\left(\frac{1}{2}\sqrt{6}, \frac{1}{2}\sqrt{6}\right)$.

Als $k = -\sqrt{6}$:

$x = -\frac{2\sqrt{6}}{4} = -\frac{1}{2}\sqrt{6}$

$y = \frac{1}{2}\sqrt{6} - \sqrt{6} = -\frac{1}{2}\sqrt{6}$

Linker raakpunt is $\left(-\frac{1}{2}\sqrt{6}, -\frac{1}{2}\sqrt{6}\right)$.

36 a Verticale asymptoot: $x = 5 \Rightarrow a = 5$

$(x-5)(y-b) = c$

$(7-5)(4-b) = c \quad \Rightarrow \quad 8 - 2b = c$

$(-1-5)(-4-b) = c \quad \Rightarrow \quad \underline{24 + 6b = c}$ _

$-16 - 8b = 0$

$-16 = 8b$

$-2 = b$

$(7-5)(4--2) = c = 12$

Vergelijking hyperbool: $(x-5)(y+2) = 12$

b $(x-5)(-1{,}99+2) > 12$

$x - 5 > 1200$

$x > 1205$

$(x-5)(-2{,}01+2) > 12$

$x - 5 < -1200$

$x < -1195$

Dus als $x < -1195$ of als $x > 1205$ is.

c $(5-a)(6-b)=c \Rightarrow 30-5b-6a+ab=c$

$(11-a)\cdot -b=c \Rightarrow -11b+ab=c$

$(-3-a)(-2-b)=c \Rightarrow 6+3b+2a+ab=c$

$$\begin{array}{ll} -11b+ab=c & -11b+ab=c \\ \underline{6+3b+2a+ab=c}\,- & \underline{30-5b-6a+ab=c}\,- \\ -14b-6-2a=0 & -6b-30+6a=0 \\ -7b-3=a & b+5=a \end{array}$$

$-7b-3=b+5$

$-8=8b$

$-1=b$

$a=b+5=-1+5=4$

$c=-11b+ab=11-4=7$

Vergelijking hyperbool: $(x-4)(y+1)=7$.

38 a $y=\frac{1}{2}(x+2)^2-2$

$y+2=\frac{1}{2}(x+2)^2$

$2y+4=(x+2)^2$ en $(x+2)^2+(y-3)^2=25$

$2y+4+(y-3)^2=25$

$y^2-4y-12=0$

$(y-6)(y+2)=0$

$y=6$ of $y=-2$

Als $y=6$:

$(x+2)^2+9=25$

$(x+2)^2=16$

$x+2=4$ of $x+2=-4$

$x=2$ of $x=-6$

Snijpunten: (2,6) en (-6,6).

Als $y=-2$:

$(x+2)^2+25=25$

$(x+2)^2=0$

$x+2=0$

$x=-2$

Snijpunt: (-2,-2).

Dus de parabool en de cirkel hebben in totaal drie snijpunten.

b $rc_{lijn}=\frac{3--1}{-7-1}=-\frac{1}{2}$

$b=-1+\frac{1}{2}=-\frac{1}{2}$

Vergelijking lijn: $y=-\frac{1}{2}x-\frac{1}{2}$.

$(x+2)^2+(-\frac{1}{2}x-\frac{1}{2}-3)^2=25$

$(x+2)^2+(-\frac{1}{2}x-3\frac{1}{2})^2=25$

$1\frac{1}{4}x^2+7\frac{1}{2}x-8\frac{3}{4}=0$

$x^2+6x-7=0$

$(x-1)(x+7)=0$

$x=1$ of $x=-7$

Als $x=1$, dan $y=-\frac{1}{2}-\frac{1}{2}=-1$.

Als $x=-7$, dan $y=3\frac{1}{2}-\frac{1}{2}=3$.

Snijpunten zijn $(1,-1)$ en $(-7,3)$.

c $y=-\frac{1}{2}x-\frac{1}{2}$ en $y=\frac{1}{2}(x+2)^2-2$

$-\frac{1}{2}x-\frac{1}{2}=\frac{1}{2}(x+2)^2-2$

$-\frac{1}{2}x-\frac{1}{2}=\frac{1}{2}x^2+2x+2-2$

$\frac{1}{2}x^2+2\frac{1}{2}x+\frac{1}{2}=0$

$x^2+5x+1=0$

$$\left.\begin{array}{l} a=1 \\ b=5 \\ c=1 \end{array}\right\} \begin{array}{l} D=25-4\cdot 1\cdot 1=21 \\ \sqrt{D}=\sqrt{21} \end{array}$$

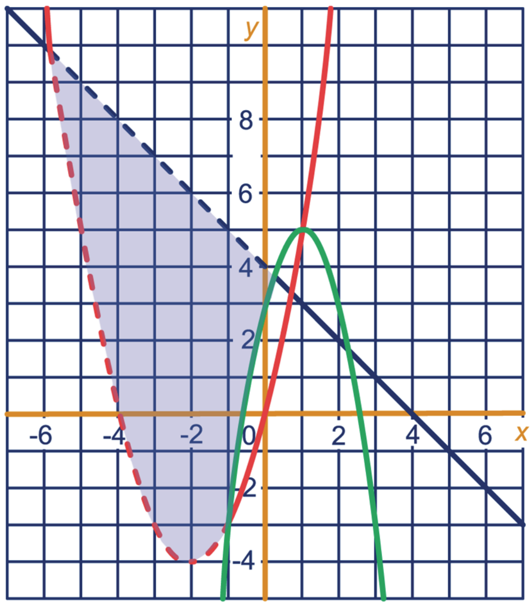
$x=\frac{-5+\sqrt{21}}{2}=-2\frac{1}{2}+\frac{1}{2}\sqrt{21}$ of $x=-2\frac{1}{2}-\frac{1}{2}\sqrt{21}$

Als $x=-2\frac{1}{2}+\frac{1}{2}\sqrt{21}$, dan $y=\frac{3}{4}-\frac{1}{4}\sqrt{21}$.

Als $x=-2\frac{1}{2}-\frac{1}{2}\sqrt{21}$, dan $y=\frac{3}{4}+\frac{1}{4}\sqrt{21}$.

Snijpunten zijn $(-2\frac{1}{2}+\frac{1}{2}\sqrt{21},\frac{3}{4}-\frac{1}{4}\sqrt{21})$ en $(-2\frac{1}{2}-\frac{1}{2}\sqrt{21},\frac{3}{4}+\frac{1}{4}\sqrt{21})$.

40



41 a Inhoud is $\pi \cdot 0{,}3^2 \cdot 8 \approx 2{,}2619$ dm³

b $c = 0{,}980 \cdot 2{,}2619 = 2{,}216662$

c $V = \pi \cdot r^2 \cdot h = \pi \cdot 0{,}3^2 \cdot h$

$p \cdot V = c$

$p \cdot \pi \cdot 0{,}3^2 \cdot h = 2{,}216662$

$p \cdot h = \frac{2{,}216662}{\pi \cdot 0{,}3^2} \approx 7{,}84$

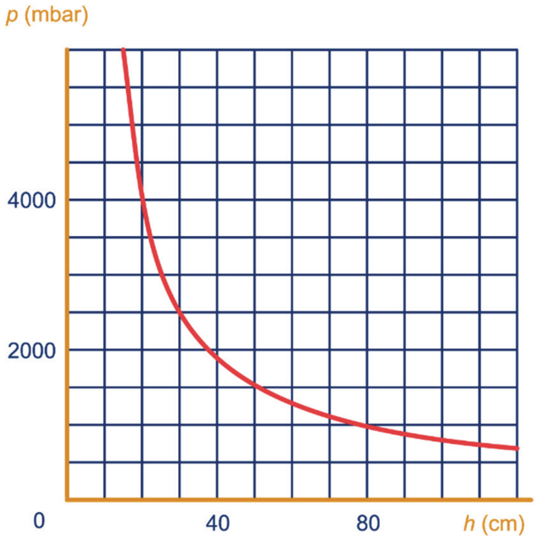
d h = 50 cm = 5 dm

$p = \frac{7{,}84}{5} = 1{,}568$ bar

e 3000 millibar = 3 bar

$h = \frac{7{,}84}{3} \approx 2{,}61$ dm

f



29.8 EXTRA OPGAVEN

1 a Parabool A:
Top (0,2) en punt (1,1).

$1 = c(1-0)^2 + 2$

$1 = c + 2$

$-1 = c$

$y = -x^2 + 2$

Parabool B:
Top (-2,3) en punt (0,1).

$1 = c(0+2)^2 + 3$

$1 = 4c + 3$

$-2 = 4c$

$-\frac{1}{2} = c$

$y = -\frac{1}{2}(x+2)^2 + 3$

Parabool C:
Top (-1,-4) en punt (0,-2).

$-2 = c(0+1)^2 - 4$

$-2 = c - 4$

$2 = c$

$y = 2(x+1)^2 - 4$

Parabool D:
Top (-1,0) en punt (0,1).

$1 = c(0+1)^2 + 0$

$1 = c$

$y = (x+1)^2$

b A en C:

$-x^2 + 2 = 2(x+1)^2 - 4$

$-x^2 + 2 = 2x^2 + 4x + 2 - 4$

$3x^2 + 4x - 4 = 0$

$\left.\begin{array}{l} a = 3 \\ b = 4 \\ c = -4 \end{array}\right\} D = 16 - 4 \cdot 3 \cdot -4 = 64, \quad \sqrt{D} = 8$

$x = \frac{-4+8}{6} = \frac{2}{3}$ of $x = \frac{-4-8}{6} = -2$

Als $x = \frac{2}{3}$, dan $y = -(\frac{2}{3})^2 + 2 = 1\frac{5}{9}$.

Als $x = -2$, dan $y = -(-2)^2 + 2 = -2$.

Snijpunten zijn $(\frac{2}{3}, 1\frac{5}{9})$ en $(-2,-2)$.

B en D:

$-\frac{1}{2}(x+2)^2 + 3 = (x+1)^2$

$-\frac{1}{2}x^2 - 2x - 2 + 3 = x^2 + 2x + 1$

$3x^2 + 8x = 0$

$3x(x + \frac{8}{3}) = 0$

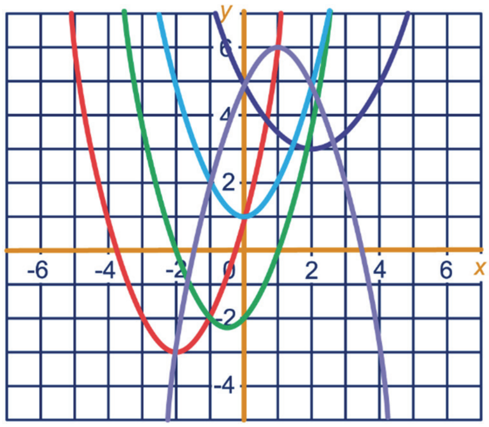
$x = 0$ of $x = -\frac{8}{3}$

Als $x = 0$, dan $y = (0+1)^2 = 1$.

Als $x = -\frac{8}{3}$, dan $y = (-\frac{8}{3} + 1)^2 = \frac{25}{9} = 2\frac{7}{9}$.

Snijpunten zijn $(0,1)$ en $(-\frac{8}{3}, 2\frac{7}{9})$.

2



3 $y = \frac{1}{4}x^2 + 3x + 2$

$y = \frac{1}{4}(x^2 + 12x + 8)$

$y = \frac{1}{4}((x+6)^2 - 36 + 8)$

$y = \frac{1}{4}(x+6)^2 - 7$

Top (-6,-7).

$y = -2x^2 + 4x + 6$

$y = -2(x^2 - 2x - 3)$

$y = -2((x-1)^2 - 1 - 3)$

$y = -2(x-1)^2 + 8$

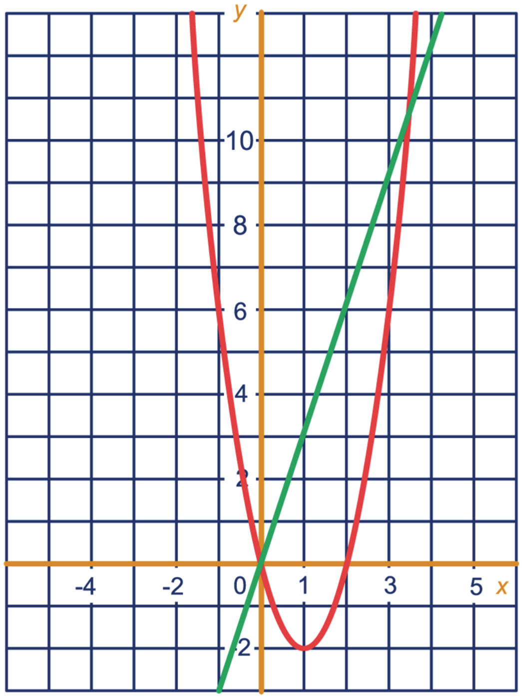
Top(1,8).

4 **a** $0 = c(2-1)^2 - 2$

$2 = c$

$y = 2(x-1)^2 - 2$

bc



d $3x = 2(x-1)^2 - 2$

$2x^2 - 7x = 0$

$2x(x - 3\frac{1}{2}) = 0$

$x = 0$ of $x = 3\frac{1}{2}$

Als $x = 0$, dan $y = 3 \cdot 0 = 0$.

Als $x = 3\frac{1}{2}$, dan $y = 3 \cdot 3\frac{1}{2} = 10\frac{1}{2}$.

Snijpunten zijn $(0,0)$ en $(3\frac{1}{2}, 10\frac{1}{2})$.

e $3x + p = 2(x-1)^2 - 2$

$2x^2 - 7x - p = 0$

$\left.\begin{array}{l} a = 2 \\ b = -7 \\ c = -p \end{array}\right\} D = 49 - 4 \cdot 2 \cdot -p = 49 + 8p$

Eén oplossing als $D = 0$.

$49 + 8p = 0$

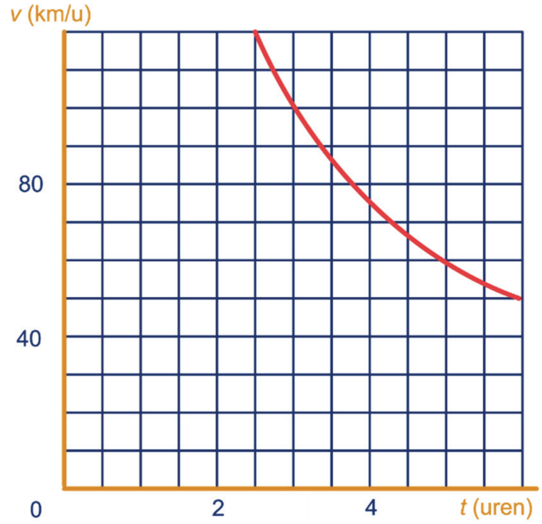
$8p = -49$

$p = -\frac{49}{8} = -6\frac{1}{8}$

5 **a** $t = 300 : 120 = 2\frac{1}{2}$ uur

b $vt = 300$

c



d Als v klein is. De grafiek daalt steeds minder snel.

6 **a** $0 = a \cdot 100 - 5 \cdot 100^2$

$50.000 = 100a$

$500 = a$

b Vanwege symmetrie wordt de grootste hoogte bereikt als $x = 50$.

Dan is $y = 500 \cdot 50 - 5 \cdot 50^2 = 12.500$, dus 12.500 meter.

7 $x^2 - 8x + 22 = 0$

$\left.\begin{array}{l} a = 1 \\ b = -8 \\ c = 22 \end{array}\right\} D = 64 - 4 \cdot 1 \cdot 22 = -24$

$D < 0$, dus geen oplossingen

$2x^2 = 5x - 3$

$2x^2 - 5x + 3 = 0$

$\left.\begin{array}{l} a = 2 \\ b = -5 \\ c = 3 \end{array}\right\} \begin{array}{l} D = 25 - 4 \cdot 2 \cdot 3 = 1 \\ \sqrt{D} = 1 \end{array}$

$x = \frac{5+1}{4} = 1\frac{1}{2}$ of $x = \frac{5-1}{4} = 1$

$3(x+2)^2 - 2x = 9$

$3x^2 + 10x + 3 = 0$

$\left.\begin{array}{l} a = 3 \\ b = 10 \\ c = 3 \end{array}\right\} \begin{array}{l} D = 100 - 4 \cdot 3 \cdot 3 = 64 \\ \sqrt{D} = 8 \end{array}$

$x = \frac{-10+8}{6} = -\frac{1}{3}$ of $x = \frac{-10-8}{6} = -3$

$-5x^2+4x-\frac{4}{5}=0$

$\left.\begin{array}{l} a=-5 \\ b=4 \\ c=-\frac{4}{5} \end{array}\right\} D=16-4\cdot -5\cdot -\frac{4}{5}=0$

$x=-\frac{4}{-10}=\frac{2}{5}$

8 a $y=-2x^2+12x$

$y=-2(x^2-6x)$

$y=-2((x-3)^2-9)$

$y=-2(x-3)^2+18$

Top (3,18).

b Voor x = 3, dan y = 18.

9 $x^2-x\sqrt{2}-4=0$

$\left.\begin{array}{l} a=1 \\ b=-\sqrt{2} \\ c=-4 \end{array}\right\} \begin{array}{l} D=2-4\cdot 1\cdot -4=18 \\ \sqrt{D}=\sqrt{18}=3\sqrt{2} \end{array}$

$x=\frac{\sqrt{2}+3\sqrt{2}}{2}=2\sqrt{2}$ of $x=\frac{\sqrt{2}-3\sqrt{2}}{2}=-\sqrt{2}$

$\sqrt{x^2+3x}=3\sqrt{2}$

$x^2+3x=18$

$x^2+3x-18=0$

$(x-3)(x+6)=0$

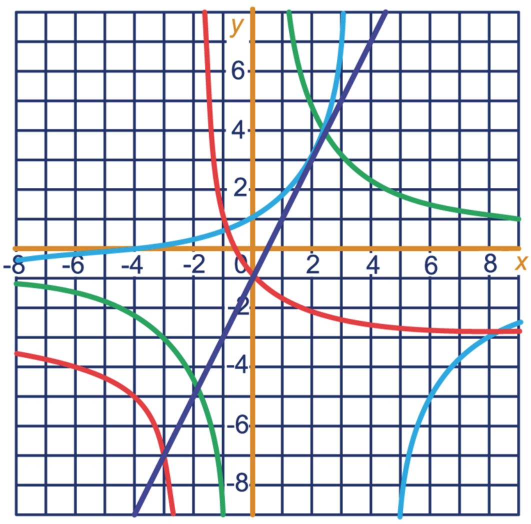
$x=3$ of $x=-6$

10 a horizontale asymptoot: y = -3
verticale asymptoot: x = -2

horizontale asymptoot: y = -1
verticale asymptoot: x = 4

horizontale asymptoot: y = 0
verticale asymptoot: x = 0

bc



d $(x+2)(y+3)=4$ en $y=2x-1$

$(x+2)(2x-1+3)=0$

$2x^2+6x=0$

$2x(x+3)=0$

$x=0$ of $x=-3$

Als $x=0$, dan $y=0-1=-1$.

Als $x=-3$, dan $y=-6-1=-7$.

Snijpunten zijn $(0,-1)$ en $(-3,-7)$.

$(x-4)(y+1)=-8$ en $y=2x-1$

$(x-4)(2x-1+1)=-8$

$2x^2-8x+8=0$

$x^2-4x+4=0$

$(x-2)^2=0$

$x=2$

Als $x=2$, dan $y=4-1=3$.

Snijpunt is $(2,3)$.

$xy=9$ en $y=2x-1$

$x(2x-1)=9$

$2x^2-x-9=0$

$\left.\begin{array}{l} a=2 \\ b=-1 \\ c=-9 \end{array}\right\} \begin{array}{l} D=1-4\cdot 2\cdot -9=73 \\ \sqrt{D}=\sqrt{73} \end{array}$

$x=\frac{1+\sqrt{73}}{4}=\frac{1}{4}+\frac{1}{4}\sqrt{73}$ of $x=\frac{1}{4}-\frac{1}{4}\sqrt{73}$

Als $x=\frac{1}{4}+\frac{1}{4}\sqrt{73}$, dan $y=-\frac{1}{2}+\frac{1}{2}\sqrt{73}$.

Als $x=\frac{1}{4}-\frac{1}{4}\sqrt{73}$, dan $y=-\frac{1}{2}-\frac{1}{2}\sqrt{73}$.

Snijpunten zijn $(\frac{1}{4}+\frac{1}{4}\sqrt{73},-\frac{1}{2}+\frac{1}{2}\sqrt{73})$ en $(\frac{1}{4}-\frac{1}{4}\sqrt{73},-\frac{1}{2}-\frac{1}{2}\sqrt{73})$.

11 horizontale asymptoot: y = 0
verticale asymptoot: x = 0
Punt op hyperbool: (3,-3)
xy = 3·-3 = -9

horizontale asymptoot: y = 1
verticale asymptoot: x = -1
Punt op hyperbool: (2,3)

$(x+1)(y-1)=c$

$(2+1)(3-1)=c$

$6=c$

$(x+1)(y-1)=6$

horizontale asymptoot: y = -3
verticale asymptoot: x = 0
Punt op hyperbool: (2,1)

$x(y+3)=c$

$2\cdot(1+3)=c$

$8=c$

$x(y+3)=8$

12 $x^2+3x+p=0$

$\left.\begin{array}{l}a=1\\b=3\\c=p\end{array}\right\}D=9-4\cdot 1\cdot p=9-4p$

Twee oplossingen als $D > 0$:

$9-4p>0$

$9>4p$

$2\frac{1}{4}>p$

Dus twee oplossingen als $p < 2\frac{1}{4}$.

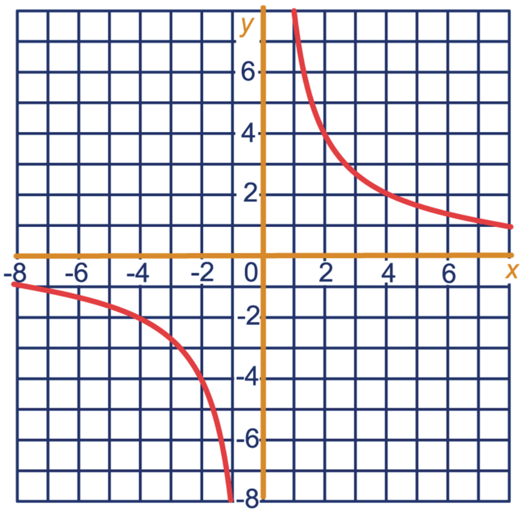
Eén oplossing als $D = 0$:

$p = 2\frac{1}{4}$

Geen oplossingen als $D < 0$:

$p > 2\frac{1}{4}$

13 a



b $xy=8$ en $y=-x+k$

$x(-x+k)=8$

$-x^2+kx-8=0$

$\left.\begin{array}{l}a=-1\\b=k\\c=-8\end{array}\right\}D=k^2-4\cdot -1\cdot -8=k^2-32$

Raken $\Rightarrow D=0$, dus

$k^2-32=0$

$k^2=32$

$k=\sqrt{32}=4\sqrt{2}$ of $k=-4\sqrt{2}$

Vergelijking raaklijnen:

$y=-x+4\sqrt{2}$ en $y=-x-4\sqrt{2}$.

14 $2(x+3)^2-4=-3x+1$

$2x^2+15x+13=0$

$\left.\begin{array}{l}a=2\\b=15\\c=13\end{array}\right\}\begin{array}{l}D=225-4\cdot 2\cdot 13=121\\ \sqrt{D}=11\end{array}$

$x=\frac{-15+11}{4}=-1$ of $x=\frac{-15-11}{4}=-6\frac{1}{2}$

Als $x=-1$, dan $y=3+1=4$.

Als $x=-6\frac{1}{2}$, dan $y=19\frac{1}{2}+1=20\frac{1}{2}$.

Snijpunten zijn $(-1,4)$ en $(-6\frac{1}{2},20\frac{1}{2})$.

15 oppervlakte driehoeken = $2\cdot x(8-x)$

oppervlakte driehoeken = $\frac{1}{4}\cdot 8\cdot 8=16$

Vergelijking:

$2\cdot x(8-x)=16$

$2x^2-16x+16=0$

$x^2-8x+8=0$

$\left.\begin{array}{l}a=1\\b=-8\\c=8\end{array}\right\}\begin{array}{l}D=64-4\cdot 1\cdot 8=32\\ \sqrt{D}=\sqrt{32}=4\sqrt{2}\end{array}$

$x=\frac{8+4\sqrt{2}}{2}=4+2\sqrt{2}$ cm of $x=4-2\sqrt{2}$ cm

Allebei de oplossingen voldoen, want $0<x<8$.

16 Oppervlakte balk is

$2(x(x+4)+x(x+3)+(x+4)(x+3))=$

$2(3x^2+14x+12)=6x^2+28x+24$.

Vergelijking:

$6x^2+28x+24=162$

$6x^2+28x-138=0$

$\left.\begin{array}{l}a=6\\b=28\\c=-138\end{array}\right\}\begin{array}{l}D=28^2-4\cdot 6\cdot -138=4096\\ \sqrt{D}=\sqrt{4096}=64\end{array}$

$x=\frac{-28+64}{12}=3$ of $x=\frac{-28-64}{12}=-7\frac{2}{3}$

Alleen $x = 3$ voldoet, omdat $x > 0$ moet zijn.

17 a $50t-5t^2=0$

$5t(10-t)=0$

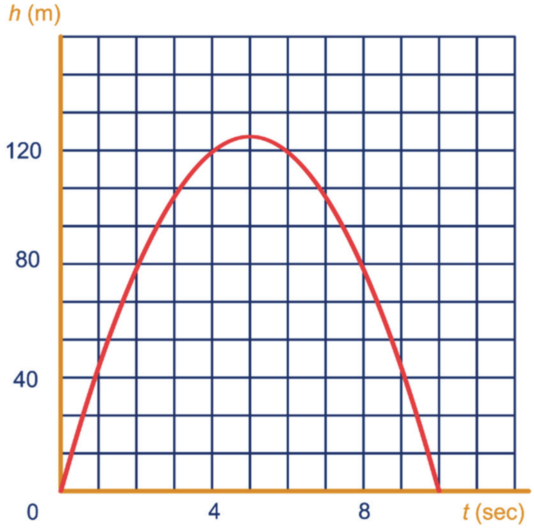
$t=0$ of $t=10$

Dus de vlucht duurt 10 – 0 = 10 sec.

b Maximale hoogte wordt bereikt na 5 sec.

$h=50\cdot 5-5\cdot 5^2=250-125=125$ m

c



d $50t - 5t^2 > 113{,}75$

$0 > 5t^2 - 50t + 113{,}75$

$t^2 - 10t + 22{,}75 < 0$

$(t - 3{,}5)(t - 6{,}5) < 0$

$3{,}5 < t < 6{,}5$

Dus tussen de 3,5 en 6,5 seconde is de hoogte van de vuurpijl meer dan 113,75 m.

18 a $(10 + 2)^2 - 10 - 10 = 124$ stippen

b $(n + 2)^2 - 2n = n^2 + 2n + 4$

c $n^2 + 2n + 4 = 10.204$

$n^2 + 2n - 10.200 = 0$

$(n - 100)(n + 102) = 0$

$n = 100$ of $n = -102$

Alleen $n = 100$ voldoet, omdat $n > 0$.

19 a $\frac{18}{45} = \frac{x}{y}$

$18y = 45x$

$y = 2\frac{1}{2}x$

Breedte van de rechthoek is

$45 - y = 45 - 2\frac{1}{2}x$.

$O = x \cdot (45 - 2\frac{1}{2}x) = 45x - 2\frac{1}{2}x^2$

b $O = 45x - 2\frac{1}{2}x^2$

$O = -2\frac{1}{2}x^2 + 45x$

$O = -2\frac{1}{2}(x^2 - 18x)$

$O = -2\frac{1}{2}((x - 9)^2 - 81)$

$O = -2\frac{1}{2}((x - 9)^2 + 202\frac{1}{2}$

De oppervlakte is maximaal als $x = 9$.

c De oppervlakte is dan $202\frac{1}{2}$.