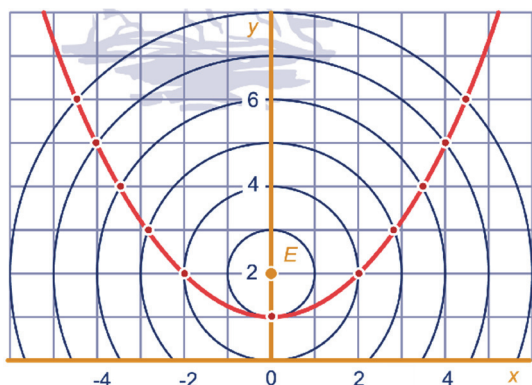


H29 PARABOLEN & HYPERBOLEN VWO

29.0 INTRO

1 ab



29.1 CONFLICTLIJN

2 a $5; \sqrt{3^2 + 4^2} = 5$

b $y; \sqrt{x^2 + (y-2)^2}$

c $y^2 = x^2 + (y-2)^2$

$$y^2 = x^2 + y^2 - 4y + 4$$

$$0 = x^2 - 4y + 4$$

$$4y = x^2 + 4$$

$$y = \frac{1}{4}x^2 + 1$$

d $3 = \frac{1}{4}x^2 + 1$

$$2 = \frac{1}{4}x^2$$

$$8 = x^2$$

$$x = \sqrt{8} \approx 2,83 \quad \text{of} \quad x = -\sqrt{8} \approx -2,83$$

$$4 = \frac{1}{4}x^2 + 1$$

$$3 = \frac{1}{4}x^2$$

$$12 = x^2$$

$$x = \sqrt{12} \approx 3,46 \quad \text{of} \quad x = -\sqrt{12} \approx -3,46$$

$$6 = \frac{1}{4}x^2 + 1$$

$$5 = \frac{1}{4}x^2$$

$$20 = x^2$$

$$x = \sqrt{20} \approx 4,47 \quad \text{of} \quad x = -\sqrt{20} \approx -4,47$$

3 a P: Afstand tot E is

$$\sqrt{\left(1\frac{1}{2}\right)^2 + \left(1\frac{3}{4}\right)^2} = \sqrt{\frac{85}{16}} = \frac{1}{4}\sqrt{85}$$

Afstand tot k is $2\frac{1}{4}$.

Q: Afstand tot E is

$$\sqrt{(-3)^2 + \left(8\frac{3}{4}\right)^2} = \sqrt{\frac{1369}{16}} = \frac{37}{4} = 9\frac{1}{4}$$

Afstand tot k is $9\frac{1}{4}$.

R: Afstand tot E is

$$\sqrt{8^2 + \left(63\frac{3}{4}\right)^2} = \sqrt{\frac{66.049}{16}} = \frac{257}{4} = 64\frac{1}{4}$$

Afstand tot k is $64\frac{1}{4}$.

Dus Q en R liggen even ver van E als van k.

b De afstand tot k is $y + \frac{1}{4}$.

De afstand tot E is $\sqrt{x^2 + \left(y - \frac{1}{4}\right)^2}$.

Dus:

$$y + \frac{1}{4} = \sqrt{x^2 + \left(y - \frac{1}{4}\right)^2} \Rightarrow$$

$$\left(y + \frac{1}{4}\right)^2 = x^2 + \left(y - \frac{1}{4}\right)^2$$

c $\left(y + \frac{1}{4}\right)^2 = x^2 + \left(y - \frac{1}{4}\right)^2$

$$y^2 + \frac{1}{2}y + \frac{1}{16} = x^2 + y^2 - \frac{1}{2}y + \frac{1}{16}$$

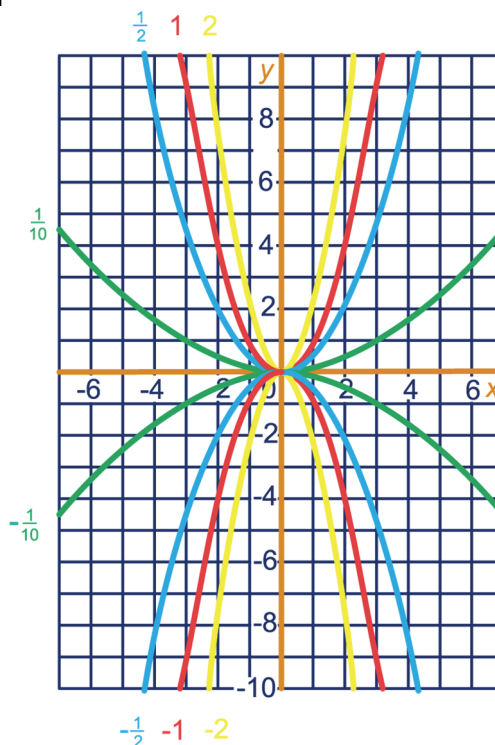
$$y = x^2$$

29.2 PARABOLEN

4 a

x	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = \frac{1}{10}x^2$	0,9	0,4	0,1	0	0,1	0,4	0,9
$y = \frac{1}{2}x^2$	4,5	2	0,5	0	0,5	2	4,5
$y = 2x^2$	18	8	2	0	2	8	18

bd



c

x	-3	-2	-1	0	1	2	3
$y = -x^2$	-9	-4	-1	0	-1	-4	-9
$y = -\frac{1}{10}x^2$	-0,9	-0,4	-0,1	0	-0,1	-0,4	-0,9
$y = -\frac{1}{2}x^2$	-4,5	-2	-0,5	0	-0,5	-2	-4,5
$y = -2x^2$	-18	-8	-2	0	-2	-8	-18

- e Dalparabool als $c > 0$,
 een bergparabool als $c < 0$.
 f Ze zijn elkaars spiegelbeeld in de x-as.
 g Dan is $y = 0$, dat is een rechte lijn, dat is de
 vergelijking van de x-as.

5

$$y = cx^2$$

$$3 = c \cdot 1^2 \quad (\text{invullen het punt } (1,3))$$

$$3 = c$$

$$y = cx^2$$

$$2 = c \cdot (-5)^2 \quad (\text{invullen het punt } (-5,2))$$

$$2 = 25c$$

$$\frac{2}{25} = c$$

$$y = cx^2$$

$$-3 = c \cdot 3^2 \quad (\text{invullen het punt } (3,-3))$$

$$-3 = 9c$$

$$-\frac{1}{3} = c$$

6

$$y = cx^2$$

$$4 = c \cdot 5^2 \quad (\text{invullen het punt } (5,4) \text{ of } (-5,4))$$

$$4 = 25c$$

$$\frac{4}{25} = c$$

7 a

$$h = cx^2$$

$$6,25 = c \cdot 10^2 \quad (\text{invullen het punt } (10 ; 6,25))$$

$$6,25 = 100c$$

$$\frac{1}{16} = c$$

Dus $h = \frac{1}{16}x^2$

b $h = \frac{1}{16} \cdot 40^2 = 100 \text{ m}$

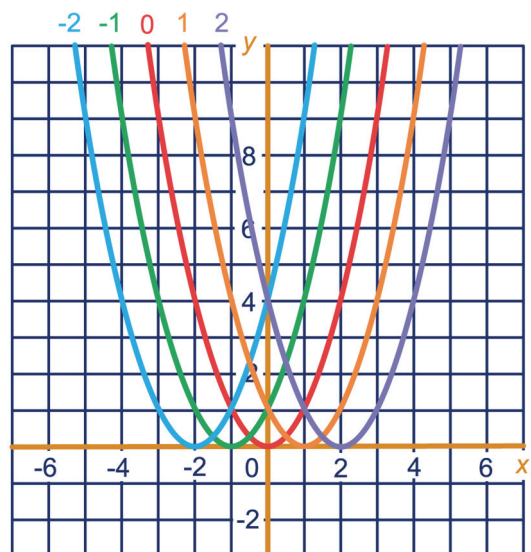
c als $x = 0$, dan $h = \frac{1}{16} \cdot 0^2 = 0 \text{ m}$
 als $x = 10$, dan $h = \frac{1}{16} \cdot 10^2 = 6,25 \text{ m}$
 als $x = 20$, dan $h = \frac{1}{16} \cdot 20^2 = 25 \text{ m}$
 als $x = 30$, dan $h = \frac{1}{16} \cdot 30^2 = 56,25 \text{ m}$
 als $x = 40$, dan $h = \frac{1}{16} \cdot 40^2 = 100 \text{ m}$

d $x = 35$, dan $h = \frac{1}{16} \cdot 35^2 = 76,5625 \text{ m}$
 De hoogte boven de Wupper is dan
 $100 - 76,5625 = 23,4375 \text{ m}$

8 a

x	-3	-2	-1	0	1	2	3
$y = x^2$	9	4	1	0	1	4	9
$y = (x-1)^2$	16	9	4	1	0	1	4

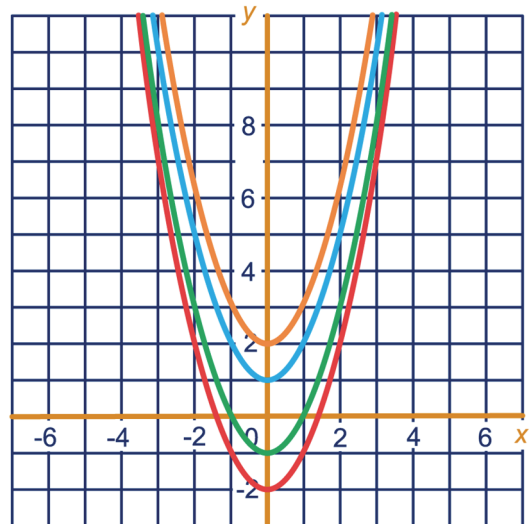
$y = (x-2)^2$	25	16	9	4	1	0	1
$y = (x+1)^2$	4	1	0	1	4	9	16
$y = (x+2)^2$	1	0	1	4	9	16	25



- b Eén eenheid naar rechts.
 c Door de grafiek twee eenheden naar links te
 schuiven.

9 a

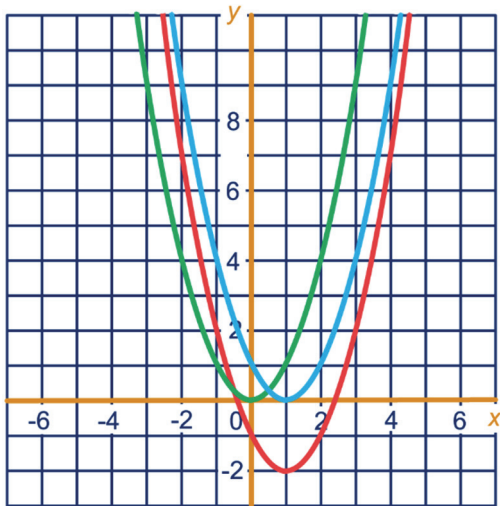
x	-3	-2	-1	0	1	2	3
$y = x^2 + 1$	10	5	2	1	2	5	10
$y = x^2 + 2$	11	6	3	2	3	6	11
$y = x^2 - 1$	8	3	0	-1	0	3	8
$y = x^2 - 2$	7	2	-1	-2	-1	2	7



b

x	-3	-2	-1	0	1	2	3
$y = (x-1)^2$	16	9	4	1	0	1	4
$y = (x-1)^2 - 2$	14	7	2	-1	-2	-1	2

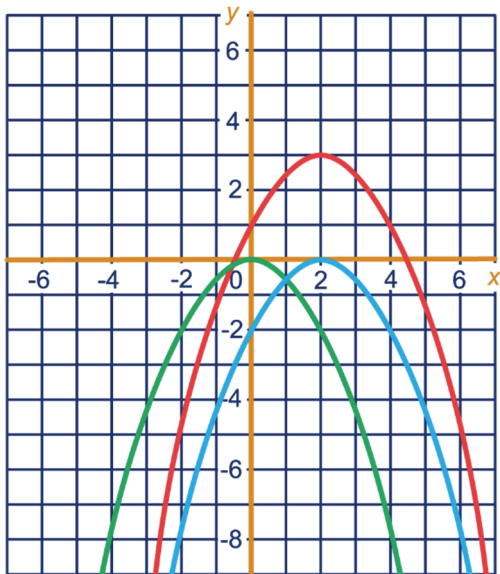
c



10 a

x	-2	-1	0	1	2	3	4
$y = -\frac{1}{2}x^2$	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2	$-4\frac{1}{2}$	-8
$y = -\frac{1}{2}(x-2)^2$	-8	$-4\frac{1}{2}$	-2	$-\frac{1}{2}$	0	$-\frac{1}{2}$	-2
$y = -\frac{1}{2}(x-2)^2 + 3$	-5	$-1\frac{1}{2}$	1	$2\frac{1}{2}$	3	$2\frac{1}{2}$	1

b



c 2; rechts
3; boven

11 a 3 eenheden naar links

b 4 eenheden naar beneden

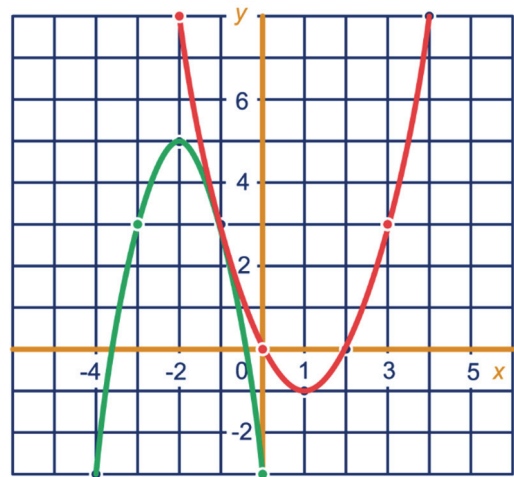
12 (2,3); (-3,-4)

13 (2,2); (-3,0)
(3,2); (0,3)

14 a (-1,2)

b $y = (2+1)^2 + 2 = 11$. Gaat niet door (2,20).
 $y = -(2+1)^2 + 2 = -7$. Gaat niet door (2,20).
 $y = 2(2+1)^2 + 2 = 20$. Gaat door (2,20).

15



16 Het punt (10,-3) ligt $10 - 4 = 6$ eenheden rechts van de symmetrieas, dus het ander punt ligt 6 eenheden links van de symmetrieas op (-2,-3). Het punt (-1,19) ligt $4 - (-1) = 5$ eenheden links van de symmetrieas, dus het ander punt ligt 5 eenheden rechts van de symmetrieas op (9,19).

29.3 VERGELIJKINGEN OPSTELLEN VOOR PARABOLEN

17 a $y = cx^2$
 $3 = c \cdot 4^2$ (invullen het punt (4,3))
 $3 = 16c$
 $\frac{3}{16} = c$

Vergelijking parabool: $y = \frac{3}{16}x^2$.

b $x = 3$ of $x = -3 \Rightarrow y = \frac{3}{16} \cdot 3^2 = 1\frac{11}{16}$

Dus $(3, 1\frac{11}{16})$ en $(-3, 1\frac{11}{16})$.

18 a $y = cx^2$
 $62,5 = c \cdot 250^2$ (invullen het punt (250 ; 62,5))
 $62,5 = 62.500c$
 $\frac{1}{1000} = c$

b $a = 500$ en $b = 0$

c $y = c(x-500)^2 + 0$
 $62,5 = c(250-500)^2$ (invullen (250 ; 62,5))
 $62,5 = 62.500c$
 $\frac{1}{1000} = c$

19 a $y = -2(x-3)^2 + 2$:
bergparabool met top (3,2), dus D.
 $y = -\frac{1}{2}(x+3)^2 + 2$:

bergparabool met top (-3,2), dus A.

b Top_B (-2,-4)
 $y = c(x+2)^2 - 4$
 $6 = c(1+2)^2 - 4$ (invullen (1,6))
 $6 = 9c - 4$
 $10 = 9c$
 $1\frac{1}{9} = c$

Vergelijking B: $y = 1\frac{1}{9}(x+2)^2 - 4$.

Top_C (3,2)

$$y = c(x - 3)^2 + 2$$

$$3 = c(5 - 3)^2 + 2 \quad (\text{invullen } (5,3))$$

$$3 = 4c + 2$$

$$1 = 4c$$

$$\frac{1}{4} = c$$

$$\text{Vergelijking C: } y = \frac{1}{4}(x - 3)^2 + 2.$$

Top_E (5,-2)

$$y = c(x - 5)^2 - 2$$

$$-4 = c(6 - 5)^2 - 2 \quad (\text{invullen } (6,-4))$$

$$-4 = c - 2$$

$$-2 = c$$

$$\text{Vergelijking E: } y = -2(x - 5)^2 - 2.$$

29.4 abc-FORMULE

20 a $y = (x + 1)^2 + 3 = x^2 + 2x + 4$

b $y = x^2 + 4x - 3 = (x + 2)^2 - 7$

c $y = 2x^2 + 8x - 6$

$$y = 2(x^2 + 4x - 3)$$

$$y = 2((x + 2)^2 - 7)$$

$$y = 2(x + 2)^2 - 14$$

d (-2,-14)

e $y = \frac{1}{2}x^2 + 3x + 2$

$$y = \frac{1}{2}(x^2 + 6x + 4)$$

$$y = \frac{1}{2}((x + 3)^2 - 5)$$

$$y = \frac{1}{2}(x + 3)^2 - 2\frac{1}{2}$$

f (-3,-2 $\frac{1}{2}$)

g $y = -x^2 + x$

$$y = -(x^2 - x)$$

$$y = -((x - \frac{1}{2})^2 - \frac{1}{4})$$

$$y = -(x - \frac{1}{2})^2 + \frac{1}{4}, \text{ Top } (\frac{1}{2}, \frac{1}{4})$$

$$y = -2x^2 + 10x + 1$$

$$y = -2(x^2 - 5x - \frac{1}{2})$$

$$y = -2((x - 2\frac{1}{2})^2 - 6\frac{3}{4})$$

$$y = -2(x - 2\frac{1}{2})^2 + 13\frac{1}{2}, \text{ Top } (2\frac{1}{2}, 13\frac{1}{2})$$

$$y = -(2x + 4)^2 + 4, \text{ Top } (-2,4)$$

$y = (x + 3)(x - 7)$, snijpunten met de x-as zijn: (-3,0) en (7,0). De symmetrieas ligt daar midden tussen.

$$\text{Symmetrieas van de parabool: } x = \frac{-3+7}{2} = 2,$$

$$y = (2 + 3)(2 - 7) = -25, \text{ Top } (2,-25).$$

21 $x = -\frac{11}{6} + \sqrt{\frac{25}{36}} = -1$ of $x = -\frac{11}{6} - \sqrt{\frac{25}{36}} = -2\frac{2}{3}$

22 a $x = -\frac{9}{2.5} + \sqrt{(\frac{9}{2.5})^2 - \frac{2}{5}}$ of $x = -\frac{9}{2.5} - \sqrt{(\frac{9}{2.5})^2 - \frac{2}{5}}$

b $x = -\frac{9}{2.5} + \sqrt{(\frac{9}{2.5})^2 - \frac{2}{5}} = \frac{9}{10} + \sqrt{\frac{41}{100}} = -\frac{9}{10} + \frac{1}{10}\sqrt{41}$ of

$$x = -\frac{9}{2.5} - \sqrt{(\frac{9}{2.5})^2 - \frac{2}{5}} = -\frac{9}{10} - \frac{1}{10}\sqrt{41}$$

23 $x = -\frac{b}{2a} + \sqrt{(\frac{b}{2a})^2 - \frac{c}{a}}$ of $x = -\frac{b}{2a} - \sqrt{(\frac{b}{2a})^2 - \frac{c}{a}}$

24 a $x = \frac{-5}{-2.2} + \sqrt{(\frac{-5}{-2.2})^2 - \frac{-25}{2}} = \frac{5}{4} + \sqrt{\frac{225}{16}} = \frac{5}{4} + \frac{15}{4} = \frac{20}{4} = 5$ of

$$x = \frac{-5}{-2.2} - \sqrt{(\frac{-5}{-2.2})^2 - \frac{-25}{2}} = \frac{5}{4} - \frac{15}{4} = -\frac{10}{4} = -2\frac{1}{2}$$

b $2 \cdot 5^2 - 5 \cdot 5 - 25 = 50 - 25 - 25 = 0$

$$2 \cdot (-2\frac{1}{2})^2 - 5 \cdot -2\frac{1}{2} - 25 = 12\frac{1}{2} + 12\frac{1}{2} - 25 = 0$$

25 a $x^2 + 4x + 5 = 0$

$$(x + 2)^2 - 4 + 5 = 0$$

$$(x + 2)^2 = -1 \text{ en dat kan voor geen enkele } x.$$

b $\sqrt{(\frac{4}{2.1})^2 - \frac{5}{1}} = \sqrt{4 - 5} = \sqrt{-1}$, maar $\sqrt{-1}$ bestaat niet.

c stap 1:

$$\text{haakjes uitwerken, } (\frac{b}{2a})^2 = \frac{b}{2a} \cdot \frac{b}{2a} = \frac{b^2}{4a^2}$$

stap 2:

$$\text{breuken gelijknamig maken, } \frac{c}{a} \cdot \frac{4a}{4a} = \frac{4ac}{4a^2}$$

stap 3:

twee breuken met dezelfde noemer optellen: de noemer zo laten en de tellers optellen.

d als $a > 0$, dan $2a \cdot 2a = 4a^2$ en

$$\text{als } a < 0, \text{ dan } -2a \cdot -2a = 4a^2$$

e als $a > 0$, $-\frac{b}{2a} + \sqrt{\frac{D}{4a^2}} = -\frac{b}{2a} + \frac{\sqrt{D}}{2a}$

$$\text{of: } -\frac{b}{2a} - \sqrt{\frac{D}{4a^2}} = -\frac{b}{2a} - \frac{\sqrt{D}}{2a}$$

$$\text{als } a < 0, \quad -\frac{b}{2a} + \sqrt{\frac{D}{4a^2}} = -\frac{b}{2a} + \frac{\sqrt{D}}{-2a} = -\frac{b}{2a} - \frac{\sqrt{D}}{2a}$$

$$\text{of: } -\frac{b}{2a} - \sqrt{\frac{D}{4a^2}} = -\frac{b}{2a} - \frac{\sqrt{D}}{-2a} = -\frac{b}{2a} + \frac{\sqrt{D}}{2a}$$

f $D = 2^2 - 4 \cdot 1 \cdot 3 = -8$

$$D = 3^2 - 4 \cdot -1 \cdot 2 = 17$$

$$D = 20^2 - 4 \cdot 4 \cdot 25 = 0$$

g geen oplossingen

$$x = \frac{-3 + \sqrt{17}}{2 \cdot -1} = 1\frac{1}{2} - \frac{1}{2}\sqrt{17} \text{ of } x = 1\frac{1}{2} + \frac{1}{2}\sqrt{17}$$

$$x = -\frac{20}{8} = -2\frac{1}{2}$$

26 Dan staat er een lineaire vergelijking.

27 $2x^2 - 3x - 35 = 0$

$$\left. \begin{array}{l} a = 2 \\ b = -3 \end{array} \right\} D = 9 - 4 \cdot 2 \cdot -35 = 289$$

$$\left. \begin{array}{l} c = -35 \end{array} \right\} \sqrt{D} = 17$$

$$x = \frac{3+17}{4} = 5 \text{ of } x = \frac{3-17}{4} = -3\frac{1}{2}$$

$$2x^2 + 4x - 1 = 0$$

$$\left. \begin{array}{l} a = 2 \\ b = 4 \\ c = -1 \end{array} \right\} \begin{array}{l} D = 16 - 4 \cdot 2 \cdot (-1) = 24 \\ \sqrt{D} = \sqrt{24} = 2\sqrt{6} \end{array}$$

$$x = \frac{-4 \pm 2\sqrt{6}}{4} = -1 \pm \frac{1}{2}\sqrt{6} \text{ of } x = \frac{-4 - 2\sqrt{6}}{4} = -1 - \frac{1}{2}\sqrt{6}$$

$$7x^2 - 6x + 2 = 0$$

$$\left. \begin{array}{l} a = 7 \\ b = -6 \\ c = 2 \end{array} \right\} D = 36 - 4 \cdot 7 \cdot 2 = -20$$

$D < 0$, dus géén oplossingen

$$\frac{1}{2}x^2 - 3x - 4\frac{1}{2} = 0$$

$$\left. \begin{array}{l} a = \frac{1}{2} \\ b = -3 \\ c = -4\frac{1}{2} \end{array} \right\} \begin{array}{l} D = 9 - 4 \cdot \frac{1}{2} \cdot (-4\frac{1}{2}) = 18 \\ \sqrt{D} = \sqrt{18} = 3\sqrt{2} \end{array}$$

$$x = \frac{3 \pm 3\sqrt{2}}{\frac{1}{2}} = 3 \pm 3\sqrt{2} \text{ of } x = \frac{3 - 3\sqrt{2}}{\frac{1}{2}} = 3 - 3\sqrt{2}$$

$$4x = 1 + 4x^2$$

$$4x^2 - 4x + 1 = 0$$

$$\left. \begin{array}{l} a = 4 \\ b = -4 \\ c = 1 \end{array} \right\} D = 16 - 4 \cdot 4 \cdot 1 = 0$$

$$x = -\frac{-4}{8} = \frac{1}{2}$$

$$(x-3)^2 = 5 - 3x$$

$$x^2 - 6x + 9 = 5 - 3x$$

$$x^2 - 3x + 4 = 0$$

$$\left. \begin{array}{l} a = 1 \\ b = -3 \\ c = 4 \end{array} \right\} D = 9 - 4 \cdot 1 \cdot 4 = -7$$

$D < 0$, dus geen oplossingen

$$5x - 3x^2 = 0$$

$$\left. \begin{array}{l} a = -3 \\ b = 5 \\ c = 0 \end{array} \right\} \begin{array}{l} D = 25 - 4 \cdot (-3) \cdot 0 = 25 \\ \sqrt{D} = 5 \end{array}$$

$$x = \frac{-5 \pm 5}{-6} = 0 \text{ of } x = \frac{-5 - 5}{-6} = \frac{10}{6} = 1\frac{2}{3}$$

28 $(x+3)^2 = 16$

$$x+3 = 4 \text{ of } x+3 = -4$$

$$x = 1 \text{ of } x = -7$$

$$(x+1)^2 = (2x+3)^2$$

$$x+1 = 2x+3 \text{ of } x+1 = -2x-3$$

$$-2 = x \text{ of } 3x = -4$$

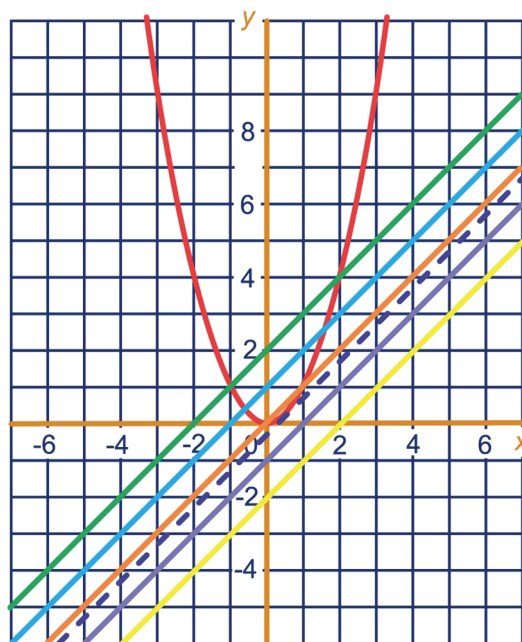
$$x = -\frac{4}{3} = -1\frac{1}{3}$$

$$x^2 + 6x + 5 = 0$$

$$(x+1)(x+5) = 0$$

$$x = -1 \text{ of } x = -5$$

29 ab



c $x^2 = x$

$$x^2 - x = 0$$

$$x(x-1) = 0$$

$$x = 0 \text{ of } x = 1$$

Snijpunten: (0,0) en (1,1)

d $x^2 = x+2$

$$x^2 - x - 2 = 0$$

$$(x-2)(x+1) = 0$$

$$x = 2 \text{ of } x = -1$$

Snijpunten: (2,4) en (-1,1)

e $x^2 = x-2$

$$x^2 - x + 2 = 0$$

$$a = 1$$

$$b = -1 \left\} D = 1 - 4 \cdot 1 \cdot 2 = -7$$

$$c = 2$$

$D < 0$, dus geen snijpunten.

f Alle lijnen hebben richtingscoëfficiënt 1.

g Zie blauwe stippellijn in opgave a.

$$\begin{aligned} \text{h } x^2 &= x - 1 \\ x^2 - x + 1 &= 0 \\ a &= 1 \\ b &= -1 \\ c &= 1 \end{aligned} \left. \vphantom{\begin{aligned} x^2 - x + 1 &= 0 \\ a &= 1 \\ b &= -1 \\ c &= 1 \end{aligned}} \right\} D = 1 - 4 \cdot 1 \cdot 1 = -3$$

$$\begin{aligned} x^2 &= x \\ x^2 - x &= 0 \\ a &= 1 \\ b &= -1 \\ c &= 0 \end{aligned} \left. \vphantom{\begin{aligned} x^2 - x &= 0 \\ a &= 1 \\ b &= -1 \\ c &= 0 \end{aligned}} \right\} D = 1 - 4 \cdot 1 \cdot 0 = 1$$

$$\begin{aligned} x^2 &= x + 1 \\ x^2 - x - 1 &= 0 \\ a &= 1 \\ b &= -1 \\ c &= -1 \end{aligned} \left. \vphantom{\begin{aligned} x^2 - x - 1 &= 0 \\ a &= 1 \\ b &= -1 \\ c &= -1 \end{aligned}} \right\} D = 1 - 4 \cdot 1 \cdot (-1) = 5$$

$$\begin{aligned} \text{i } x^2 - x - k &= 0 \\ a &= 1 \\ b &= -1 \\ c &= -k \end{aligned} \left. \vphantom{\begin{aligned} x^2 - x - k &= 0 \\ a &= 1 \\ b &= -1 \\ c &= -k \end{aligned}} \right\} D = 1 - 4 \cdot 1 \cdot (-k) = 1 + 4k$$

$$\begin{aligned} \text{j } 1 + 4k &= 0 \\ 4k &= -1 \\ k &= -\frac{1}{4} \end{aligned}$$

$$\begin{aligned} \text{k } x^2 &= x - \frac{1}{4} \\ x^2 - x + \frac{1}{4} &= 0 \\ x &= -\frac{-1}{2} = \frac{1}{2} \Rightarrow y = \left(\frac{1}{2}\right)^2 = \frac{1}{4} \\ \text{Raakpunt is } &\left(\frac{1}{2}, \frac{1}{4}\right). \end{aligned}$$

$$\begin{aligned} \text{30 a } -x^2 + 1 &= ax + 3 \\ x^2 + ax + 2 &= 0 \end{aligned}$$

$$\begin{aligned} \text{b } x^2 + ax + 2 &= 0 \\ a &= 1 \\ b &= a \\ c &= 2 \end{aligned} \left. \vphantom{\begin{aligned} x^2 + ax + 2 &= 0 \\ a &= 1 \\ b &= a \\ c &= 2 \end{aligned}} \right\} D = a^2 - 4 \cdot 1 \cdot 2 = a^2 - 8$$

$$\begin{aligned} \text{c Raken } \Rightarrow D &= 0, \text{ dus} \\ a^2 - 8 &= 0 \\ a^2 &= 8 \\ a &= \sqrt{8} = 2\sqrt{2} \text{ of } a = -\sqrt{8} = -2\sqrt{2} \end{aligned}$$

$$\begin{aligned} \text{d } x^2 + 2\sqrt{2}x + 2 &= 0 \\ x &= -\frac{2\sqrt{2}}{2} = -\sqrt{2} \Rightarrow y = -(-\sqrt{2})^2 + 1 = -1 \\ \text{Raakpunt is } &(-\sqrt{2}, -1). \\ x^2 - 2\sqrt{2}x + 2 &= 0 \\ x &= -\frac{-2\sqrt{2}}{2} = \sqrt{2} \Rightarrow y = -(\sqrt{2})^2 + 1 = -1 \\ \text{Raakpunt is } &(\sqrt{2}, -1). \end{aligned}$$

$$\begin{aligned} \text{31 a } x^2 + 5x + 4 &= 0 \\ (x + 1)(x + 4) &= 0 \\ x &= -1 \text{ of } x = -4 \end{aligned}$$

$$\begin{aligned} \text{b } x^2 + kx + 4 &= 0 \\ a &= 1 \\ b &= k \\ c &= 4 \end{aligned} \left. \vphantom{\begin{aligned} x^2 + kx + 4 &= 0 \\ a &= 1 \\ b &= k \\ c &= 4 \end{aligned}} \right\} D = k^2 - 4 \cdot 1 \cdot 4 = k^2 - 16$$

Eén oplossing, dus $D = 0$.

$$\begin{aligned} k^2 - 16 &= 0 \\ k^2 &= 16 \\ k &= 4 \text{ of } k = -4 \end{aligned}$$

$$\begin{aligned} \text{c } x^2 + 4x + 4 &= 0 \\ (x + 2)^2 &= 0 \\ x &= -2 \end{aligned}$$

32 a x-as $\Rightarrow y = 0$, dan:

$$\begin{aligned} px^2 - 6x - 1 &= 0 \\ a &= p \\ b &= -6 \\ c &= -1 \end{aligned} \left. \vphantom{\begin{aligned} px^2 - 6x - 1 &= 0 \\ a &= p \\ b &= -6 \\ c &= -1 \end{aligned}} \right\} D = 36 - 4 \cdot p \cdot (-1) = 36 + 4p$$

Eén raakpunt met x-as $\Rightarrow D = 0$

$$\begin{aligned} 36 + 4p &= 0 \\ 4p &= -36 \\ p &= -9 \end{aligned}$$

b x-as $\Rightarrow y = 0$, dan:

$$\begin{aligned} \frac{1}{2}x^2 - px + 2 &= 0 \\ a &= \frac{1}{2} \\ b &= -p \\ c &= 2 \end{aligned} \left. \vphantom{\begin{aligned} \frac{1}{2}x^2 - px + 2 &= 0 \\ a &= \frac{1}{2} \\ b &= -p \\ c &= 2 \end{aligned}} \right\} D = (-p)^2 - 4 \cdot \frac{1}{2} \cdot 2 = p^2 - 4$$

Twee snijpunten met x-as $\Rightarrow D > 0$

$$\begin{aligned} p^2 - 4 &> 0 \\ p^2 &> 4 \\ p &< -2 \text{ of } p > 2 \end{aligned}$$

c x-as $\Rightarrow y = 0$, dan:

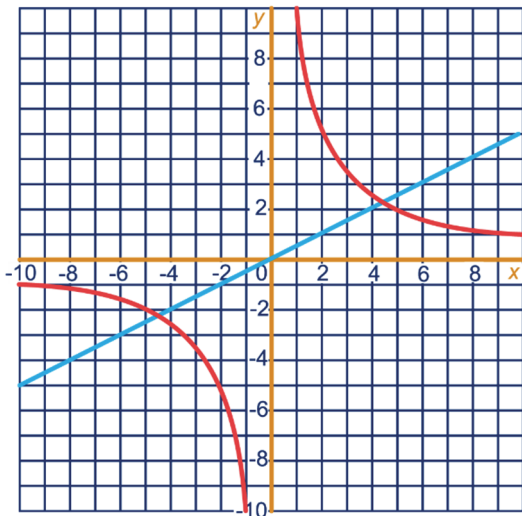
$$\begin{aligned} 2x^2 + 4x + p &= 0 \\ a &= 2 \\ b &= 4 \\ c &= p \end{aligned} \left. \vphantom{\begin{aligned} 2x^2 + 4x + p &= 0 \\ a &= 2 \\ b &= 4 \\ c &= p \end{aligned}} \right\} D = 16 - 4 \cdot 2 \cdot p = 16 - 8p$$

Geen snij- of raakpunten met x-as $\Rightarrow D < 0$

$$\begin{aligned} 16 - 8p &< 0 \\ 16 &< 8p \\ 2 &< p \end{aligned}$$

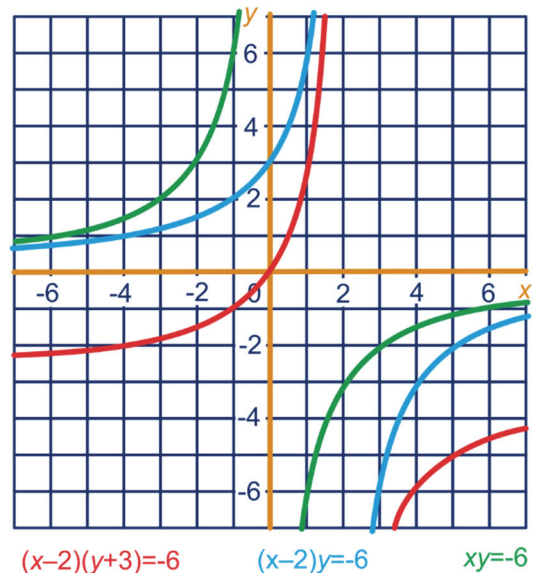
29.5 HYPERBOLEN

33 a Zie de twee rode lijnen hieronder.

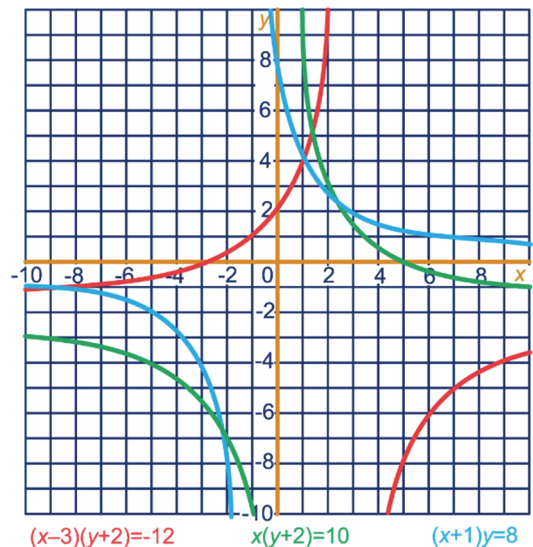


- b $-1\frac{1}{2} \cdot -7 = 10\frac{1}{2}$, ligt er niet op
 $\sqrt{20} \cdot \sqrt{5} = \sqrt{100} = 10$, ligt er op
 $-\sqrt{10} \cdot -\sqrt{10} = \sqrt{100} = 10$, ligt er op
 $8 \cdot 1\frac{1}{4} = 10$, ligt er op
- c $10 : \frac{5}{3} = 6$
 $10 : -2\frac{1}{2} = -4$
 $10 : 2\sqrt{5} = \sqrt{5}$
 $10 : -5\sqrt{2} = -\sqrt{2}$
- d $(\sqrt{10}, \sqrt{10})$ en $(-\sqrt{10}, -\sqrt{10})$
- e Zie blauwe lijn hierboven.
- f $y = \frac{1}{2}x$
- g $x \cdot \frac{1}{2}x = 10$
 $\frac{1}{2}x^2 = 10$
 $x^2 = 20$
 $x = \sqrt{20} = 2\sqrt{5}$ of $x = -\sqrt{20} = -2\sqrt{5}$
 Als $x = 2\sqrt{5}$, dan $y = \frac{1}{2}x = \sqrt{5}$.
 Als $x = -2\sqrt{5}$, dan $y = -\sqrt{5}$.
 Snijpunten zijn $(2\sqrt{5}, \sqrt{5})$ en $(-2\sqrt{5}, -\sqrt{5})$.
- h $10 : 4 = 2\frac{1}{2} = 2,5$
 $10 : 40 = \frac{1}{4} = 0,25$
 $10 : 400 = \frac{1}{40} = 0,025$
 $10 : 4000 = \frac{1}{400} = 0,0025$
- i De y-as, en de vergelijking van de y-as is $x = 0$.

34 a b e



- c twee eenheden naar rechts
d horizontale asymptoot: $y = 0$
 verticale asymptoot: $x = 2$
f horizontale asymptoot: $y = -3$
 verticale asymptoot: $x = 2$
g drie eenheden naar beneden
- 35 a horizontale asymptoot: $y = -2$
 verticale asymptoot: $x = 0$
- horizontale asymptoot: $y = -2$
 verticale asymptoot: $x = 3$
- horizontale asymptoot: $y = 0$
 verticale asymptoot: $x = -1$
- b



- 36 a $(x+1)(y-3) = 12$
 horizontale asymptoot: $y = 3$
 verticale asymptoot: $x = -1$
 Dus A.

$$(x - 2)(y + 4) = -8$$

horizontale asymptoot: $y = -4$

verticale asymptoot: $x = 2$

Dus D.

b horizontale asymptoot B: $y = 2$

verticale asymptoot B: $x = -1$

$$(x + 1)(y - 2) = c$$

$$(-5 + 1)(6 - 2) = c \quad (\text{invullen } (-5, 6))$$

$$-16 = c$$

$$\text{Vergelijking B: } (x + 1)(y - 2) = -16.$$

horizontale asymptoot C: $y = 0$

verticale asymptoot C: $x = 0$

$$xy = c$$

$$-4 \cdot -6 = c \quad (\text{invullen } (-4, -6))$$

$$24 = c$$

$$\text{Vergelijking C: } xy = 24.$$

horizontale asymptoot E: $y = 2$

verticale asymptoot E: $x = 4$

$$(x - 4)(y - 2) = c$$

$$(7 - 4)(4 - 2) = c \quad (\text{invullen } (7, 4))$$

$$6 = c$$

$$\text{Vergelijking E: } (x - 4)(y - 2) = 6.$$

Vergelijking:

$$x\left(\frac{2}{3}x + 2\right) = 12$$

$$\frac{2}{3}x^2 + 2x - 12 = 0$$

$$x^2 + 3x - 18 = 0$$

$$(x - 3)(x + 6) = 0$$

$$x = 3 \quad \text{of} \quad x = -6$$

$$\text{Als } x = 3, \text{ dan } y = \frac{12}{3} = 4.$$

$$\text{Als } x = -6, \text{ dan } y = \frac{12}{-6} = -2.$$

Snijpunten zijn $(3, 4)$ en $(-6, -2)$.

e $x(-x + k) = 12$

$$-x^2 + kx - 12 = 0$$

$$x^2 - kx + 12 = 0$$

$$a = 1$$

$$b = -k \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} D = (-k)^2 - 4 \cdot 1 \cdot 12 = k^2 - 48$$

$$c = 12$$

$$k^2 - 48 = 0$$

$$k^2 = 48$$

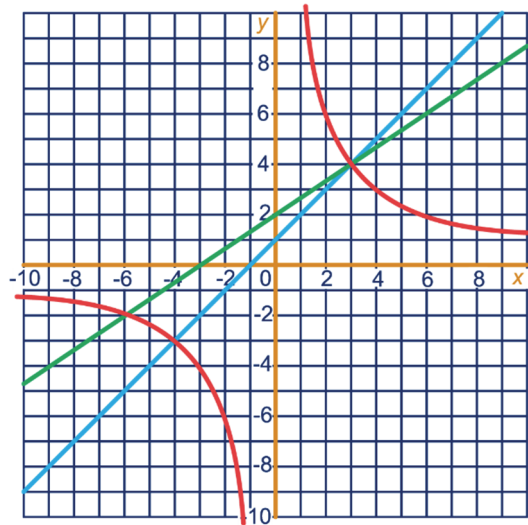
$$k = \sqrt{48} = 4\sqrt{3} \quad \text{of} \quad k = -4\sqrt{3}$$

Vergelijkingen van de raaklijnen:

$$y = -x + 4\sqrt{3} \quad \text{en} \quad y = -x - 4\sqrt{3}.$$

29.6 GEMENGDE OPGAVEN

37 ac



b $x(x + 1) = 12$

$$x^2 + x - 12 = 0$$

$$(x - 3)(x + 4) = 0$$

$$x = 3 \quad \text{of} \quad x = -4$$

$$\text{Als } x = 3, \text{ dan } y = 3 + 1 = 4.$$

$$\text{Als } x = -4, \text{ dan } y = -4 + 1 = -3.$$

Snijpunten zijn $(3, 4)$ en $(-4, -3)$.

d $2x - 3y + 6 = 0$

$$2x + 6 = 3y$$

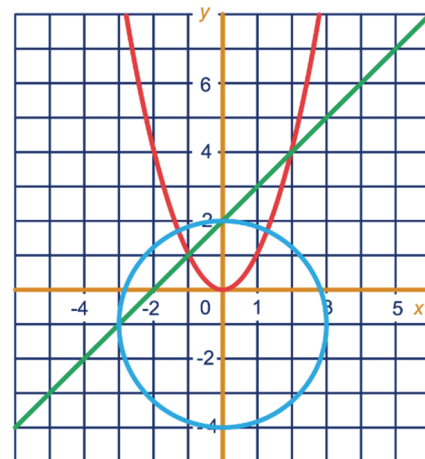
$$\frac{2}{3}x + 2 = y$$

38 a $x^2 + y^2 + 2y = 8$

$$x^2 + (y + 1)^2 - 1 = 8$$

$$x^2 + (y + 1)^2 = 9$$

bc



d $x - y + 2 = 0 \Leftrightarrow x + 2 = y$ en $x^2 + (y + 1)^2 = 9$

Vergelijking:

$$x^2 + (x + 2 + 1)^2 = 9$$

$$x^2 + (x + 3)^2 = 9$$

$$2x^2 + 6x = 0$$

$$2x(x + 3) = 0$$

$$x = 0 \quad \text{of} \quad x = -3$$

$$\text{Als } x = 0, \text{ dan } y = 0 + 2 = 2.$$

Als $x = -3$, dan $y = -3 + 2 = -1$.

Snijpunten zijn $(0,2)$ en $(-6,-2)$.

e $x + 2 = y$ en $y = x^2$

$$x + 2 = x^2$$

$$x^2 - x - 2 = 0$$

$$(x - 2)(x + 1) = 0$$

$$x = 2 \text{ of } x = -1$$

Als $x = 2$, dan $y = 2 + 2 = 4$.

Als $x = -1$, dan $y = -1 + 2 = 1$.

Snijpunten zijn $(2,4)$ en $(-1,1)$.

f $y = x^2$ en $x^2 + (y + 1)^2 = 9$

$$y + (y + 1)^2 = 9$$

$$y^2 + 3y - 8 = 0$$

$$\left. \begin{array}{l} a = 1 \\ b = 3 \\ c = -8 \end{array} \right\} \begin{array}{l} D = 9 - 4 \cdot 1 \cdot -8 = 41 \\ \sqrt{D} = \sqrt{41} \end{array}$$

$$y = \frac{-3 + \sqrt{41}}{2} = -1\frac{1}{2} + \frac{1}{2}\sqrt{41} \text{ of } y = -1\frac{1}{2} - \frac{1}{2}\sqrt{41}$$

Alleen $y = -1\frac{1}{2} + \frac{1}{2}\sqrt{41}$ voldoet, want $y \geq 0$.

39 a $y = -x^2 + 2x + 3$

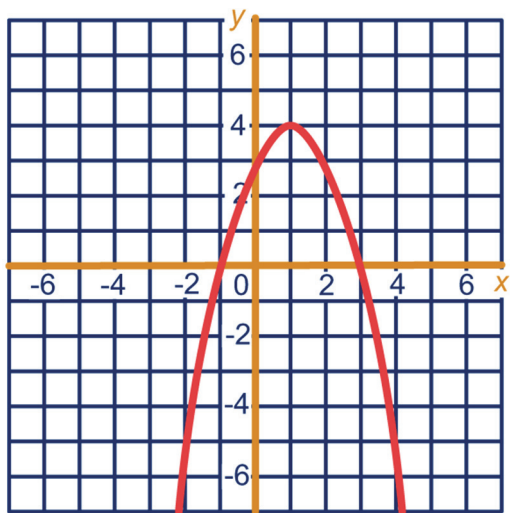
$$y = -(x^2 - 2x - 3)$$

$$y = -((x - 1)^2 - 4)$$

$$y = -(x - 1)^2 + 4$$

Top $(1,4)$.

b



c $1; 4$

d $y \leq 4$

40 a $-x^2 + 4x + 5 = 0$

$$x^2 - 4x - 5 = 0$$

$$(x - 5)(x + 1) = 0$$

$$x = 5 \text{ of } x = -1$$

Snijpunten: $(5,0)$ en $(-1,0)$.

b Vergelijking symmetrieas: $x = \frac{5-1}{2} = 2$.

c $x = 2 \Rightarrow y = -2^2 + 4 \cdot 2 + 5 = -4 + 8 + 5 = 9$
Top $(2,9)$.

41 a $p \cdot V = c$

$$4 \cdot 5 = c$$

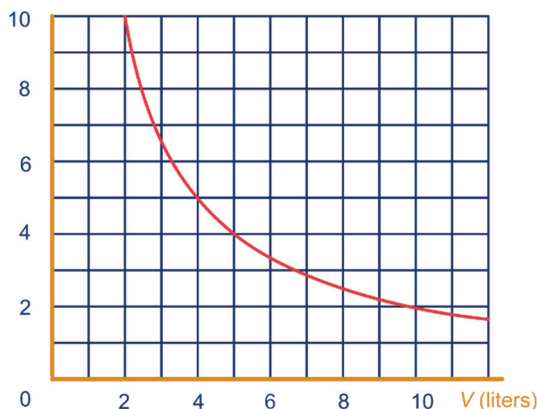
$$20 = c$$

$$p \cdot 3 = 20$$

$$p = \frac{20}{3} = 6\frac{2}{3} \text{ bar}$$

b

p (bar)



c $V > \frac{20}{3} = 6\frac{2}{3} \text{ bar}$

42 a $C = -20x^2 + 60x$

$$C = -20(x^2 - 3x)$$

$$C = -20((x - 1\frac{1}{2})^2 - 2\frac{1}{4})$$

$$C = -20(x - 1\frac{1}{2})^2 + 45$$

Top $(1\frac{1}{2}, 45)$.

b Een bergparabool.

c Bij hoogte $1\frac{1}{2}$, de capaciteit is dan 45.

43 parabool
cirkel
lijn
verticale lijn
hyperbool

SUPER OPGAVEN

14 y vervangen in de vergelijking

$$x + y + 6 = (x - y + 3)^2 \text{ door } x + 3. \text{ Dus:}$$

$$x + (x + 3) + 6 = (x - (x + 3) + 3)^2$$

$$2x + 9 = 0$$

$$x = -4\frac{1}{2} \Rightarrow y = -4\frac{1}{2} + 3 = -1\frac{1}{2}$$

Top $(-4\frac{1}{2}, -1\frac{1}{2})$.

16 Als de top op de y -as ligt, dan zijn $(-2,4)$ en $(3,6)$ ook punten van de parabool.

Dus dan moet het een dalparabool zijn.

19 a $y = c(x-4)^2 + 6$ (invullen top $(a,b) = (4,6)$)
 $3 = c(1-4)^2 + 6$ (invullen punt $(x,y) = (1,3)$)
 $3 = 9c + 6$
 $-3 = 9c$
 $-\frac{1}{3} = c$

Vergelijking parabool: $y = -\frac{1}{3}(x-4)^2 + 6$.

b Top op de y -as $\Rightarrow a = 0 \Rightarrow y = cx^2 + b$
 $9 = c \cdot (-3)^2 + b \Rightarrow 9 = 9c + b$
 $-3 = c \cdot 2^2 + b \Rightarrow \underline{-3 = 4c + b}$
 $12 = 5c$
 $2\frac{2}{5} = \frac{12}{5} = c$

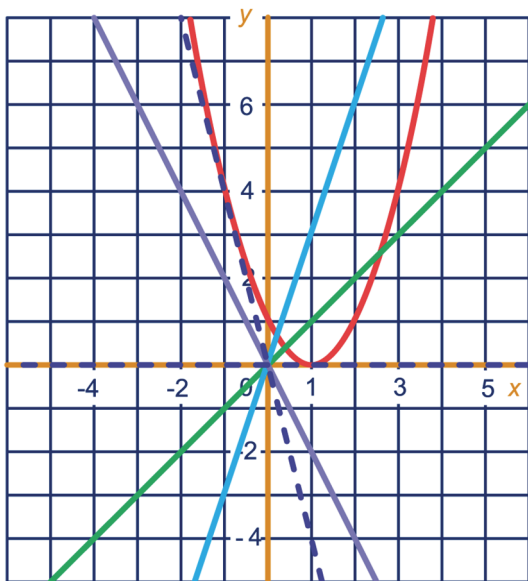
$b = 9 - 9 \cdot 2\frac{2}{5} = -12\frac{3}{5}$

Vergelijking parabool: $y = 2\frac{2}{5}x^2 - 12\frac{3}{5}$.

c Top op de x -as $\Rightarrow b = 0 \Rightarrow y = c(x-a)^2$
 $y = \frac{1}{3}(x-a)^2$ invullen $\frac{1}{3}$ voor c
 $3 = \frac{1}{3}(4-a)^2$ invullen het punt $(4,3)$
 $9 = (4-a)^2$
 $4-a = 3$ of $4-a = -3$
 $a = 1$ of $a = 7$

Vergelijking symmetrieas: $x = 1$ of $x = 7$.

29 aef



b Omdat $0 = k \cdot 0$ klopt, wat je ook voor k neemt.

c Als $k = 0$, dan $y = 0$. ; $5 = k \cdot 2 \Rightarrow k = -2\frac{1}{2}$

d De verticale as, dus de y -as.

f Zie de twee stippellijnen in opgave a.

g $(x-1)^2 = kx$
 $x^2 - 2x + 1 = kx$
 $x^2 - 2x - kx + 1 = 0$
 $x^2 - (2+k)x + 1 = 0$

h $D = (2+k)^2 - 4 \cdot 1 \cdot 1 = (2+k)^2 - 4$
i Als $k = 1$, dan $D = 9 - 4 = 5$, dus twee snijpunten.
j $(2+k)^2 - 4 = 0$
 $(2+k)^2 = 4$
 $2+k = 2$ of $2+k = -2$
 $k = 0$ of $k = -4$
k $y = 0$ en $y = -4x$

31 a $x^2 + (-x+k)^2 = 3$
 $2x^2 - 2kx + k^2 - 3 = 0$
 $a = 2$
 $b = -2k$
 $c = k^2 - 3$
 $D = (-2k)^2 - 4 \cdot 2 \cdot (k^2 - 3) = 24 - 4k^2$

b $24 - 4k^2 = 0$
 $24 = 4k^2$
 $6 = k^2$
 $k = \sqrt{6}$ of $k = -\sqrt{6}$

Als $k = \sqrt{6}$:

$x = -\frac{b}{2a} = -\frac{-2\sqrt{6}}{4} = \frac{1}{2}\sqrt{6}$

$y = -\frac{1}{2}\sqrt{6} + \sqrt{6} = \frac{1}{2}\sqrt{6}$

Rechter raakpunt is $(\frac{1}{2}\sqrt{6}, \frac{1}{2}\sqrt{6})$.

Als $k = -\sqrt{6}$:

$x = -\frac{2\sqrt{6}}{4} = -\frac{1}{2}\sqrt{6}$

$y = \frac{1}{2}\sqrt{6} - \sqrt{6} = -\frac{1}{2}\sqrt{6}$

Linker raakpunt is $(-\frac{1}{2}\sqrt{6}, -\frac{1}{2}\sqrt{6})$.

36 a Verticale asymptoot: $x = 5 \Rightarrow a = 5$
 $(x-5)(y-b) = c$
 $(7-5)(4-b) = c \Rightarrow 8 - 2b = c$
 $(-1-5)(-4-b) = c \Rightarrow \underline{24 + 6b = c}$
 $-16 - 8b = 0$
 $-16 = 8b$
 $-2 = b$

$(7-5)(4-(-2)) = c = 12$

Vergelijking hyperbool: $(x-5)(y+2) = 12$

b $(x-5)(-1,99+2) > 12$
 $x-5 > 1200$
 $x > 1205$
 $(x-5)(-2,01+2) > 12$
 $x-5 < -1200$
 $x < -1195$

Dus als $x < -1195$ of als $x > 1205$ is.

$$\begin{aligned}
\text{c } (5-a)(6-b) &= c &\Rightarrow 30-5b-6a+ab &= c \\
(11-a) \cdot -b &= c &\Rightarrow -11b+ab &= c \\
(-3-a)(-2-b) &= c &\Rightarrow 6+3b+2a+ab &= c \\
-11b+ab &= c &-11b+ab &= c \\
\frac{6+3b+2a+ab}{-14b-6-2a} &= \frac{c}{0} &\frac{30-5b-6a+ab}{-6b-30+6a} &= \frac{c}{0} \\
-7b-3 &= a &b+5 &= a \\
-7b-3 &= b+5 && \\
-8 &= 8b && \\
-1 &= b && \\
a &= b+5 = -1+5 = 4 && \\
c &= -11b+ab = 11-4 = 7 && \\
\text{Vergelijking hyperbool: } &(x-4)(y+1) &= 7. &
\end{aligned}$$

$$\begin{aligned}
\text{38 a } y &= \frac{1}{2}(x+2)^2 - 2 \\
y+2 &= \frac{1}{2}(x+2)^2 \\
2y+4 &= (x+2)^2 \text{ en } (x+2)^2 + (y-3)^2 = 25
\end{aligned}$$

$$\begin{aligned}
2y+4+(y-3)^2 &= 25 \\
y^2-4y-12 &= 0 \\
(y-6)(y+2) &= 0 \\
y &= 6 \text{ of } y = -2
\end{aligned}$$

$$\begin{aligned}
\text{Als } y &= 6: \\
(x+2)^2+9 &= 25 \\
(x+2)^2 &= 16 \\
x+2 &= 4 \text{ of } x+2 = -4 \\
x &= 2 \text{ of } x = -6 \\
\text{Snijpunten: } &(2,6) \text{ en } (-6,6).
\end{aligned}$$

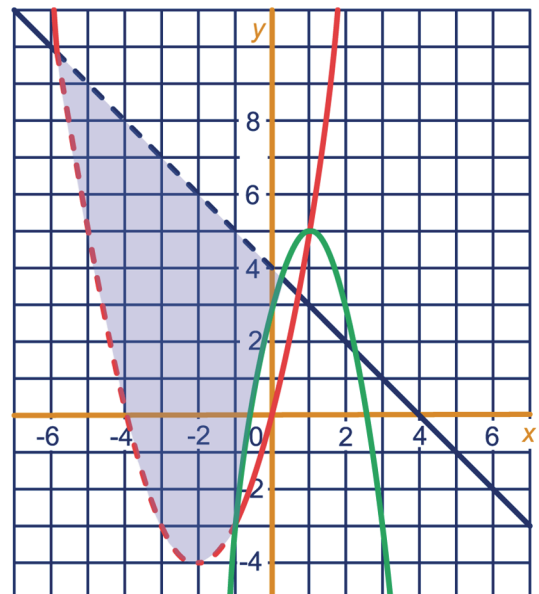
$$\begin{aligned}
\text{Als } y &= -2: \\
(x+2)^2+25 &= 25 \\
(x+2)^2 &= 0 \\
x+2 &= 0 \\
x &= -2 \\
\text{Snijpunt: } &(-2,-2). \\
\text{Dus de parabool en de cirkel hebben in totaal} & \\
\text{drie snijpunten.} &
\end{aligned}$$

$$\begin{aligned}
\text{b } r_{\text{lijn}} &= \frac{3-1}{-7-1} = -\frac{1}{2} \\
b &= -1 + \frac{1}{2} = -\frac{1}{2} \\
\text{Vergelijking lijn: } &y = -\frac{1}{2}x - \frac{1}{2}.
\end{aligned}$$

$$\begin{aligned}
(x+2)^2 + (-\frac{1}{2}x - \frac{1}{2} - 3)^2 &= 25 \\
(x+2)^2 + (-\frac{1}{2}x - 3\frac{1}{2})^2 &= 25 \\
1\frac{1}{4}x^2 + 7\frac{1}{2}x - 8\frac{3}{4} &= 0 \\
x^2 + 6x - 7 &= 0 \\
(x-1)(x+7) &= 0 \\
x &= 1 \text{ of } x = -7 \\
\text{Als } x &= 1, \text{ dan } y = -\frac{1}{2} - \frac{1}{2} = -1. \\
\text{Als } x &= -7, \text{ dan } y = 3\frac{1}{2} - \frac{1}{2} = 3. \\
\text{Snijpunten zijn } &(1,-1) \text{ en } (-7,3).
\end{aligned}$$

$$\begin{aligned}
\text{c } y &= -\frac{1}{2}x - \frac{1}{2} \text{ en } y = \frac{1}{2}(x+2)^2 - 2 \\
-\frac{1}{2}x - \frac{1}{2} &= \frac{1}{2}(x+2)^2 - 2 \\
-\frac{1}{2}x - \frac{1}{2} &= \frac{1}{2}x^2 + 2x + 2 - 2 \\
\frac{1}{2}x^2 + 2\frac{1}{2}x + \frac{1}{2} &= 0 \\
x^2 + 5x + 1 &= 0 \\
\left. \begin{aligned} a &= 1 \\ b &= 5 \\ c &= 1 \end{aligned} \right\} D &= 25 - 4 \cdot 1 \cdot 1 = 21 \\
& \sqrt{D} = \sqrt{21} \\
x &= \frac{-5 \pm \sqrt{21}}{2} = -2\frac{1}{2} \pm \frac{1}{2}\sqrt{21} \text{ of } x = -2\frac{1}{2} - \frac{1}{2}\sqrt{21} \\
\text{Als } x &= -2\frac{1}{2} + \frac{1}{2}\sqrt{21}, \text{ dan } y = \frac{3}{4} - \frac{1}{4}\sqrt{21}. \\
\text{Als } x &= -2\frac{1}{2} - \frac{1}{2}\sqrt{21}, \text{ dan } y = \frac{3}{4} + \frac{1}{4}\sqrt{21}. \\
\text{Snijpunten zijn } &(-2\frac{1}{2} + \frac{1}{2}\sqrt{21}, \frac{3}{4} - \frac{1}{4}\sqrt{21}) \text{ en } \\
&(-2\frac{1}{2} - \frac{1}{2}\sqrt{21}, \frac{3}{4} + \frac{1}{4}\sqrt{21}).
\end{aligned}$$

40



41 a Inhoud is $\pi \cdot 0,3^2 \cdot 8 \approx 2,2619 \text{ dm}^3$

b $c = 0,980 \cdot 2,2619 = 2,216662$

c $V = \pi \cdot r^2 \cdot h = \pi \cdot 0,3^2 \cdot h$

$$p \cdot V = c$$

$$p \cdot \pi \cdot 0,3^2 \cdot h = 2,216662$$

$$p \cdot h = \frac{2,216662}{\pi \cdot 0,3^2} \approx 7,84$$

d $h = 50 \text{ cm} = 5 \text{ dm}$

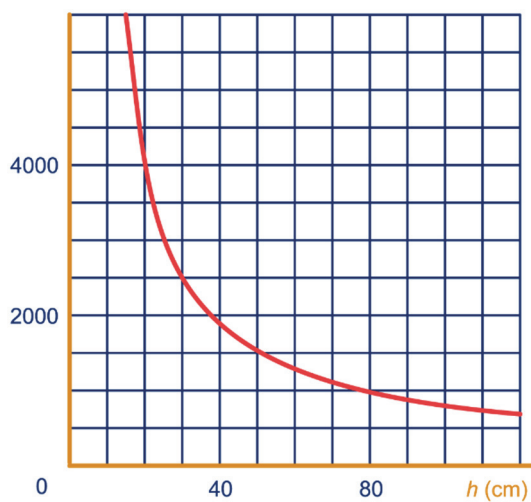
$$p = \frac{7,84}{5} = 1,568 \text{ bar}$$

e 3000 millibar = 3 bar

$$h = \frac{7,84}{3} \approx 2,61 \text{ dm}$$

f

p (mbar)



29.8 EXTRA OPGAVER

1 a Parabool A:

Top (0,2) en punt (1,1).

$$1 = c(1-0)^2 + 2$$

$$1 = c + 2$$

$$-1 = c$$

$$y = -x^2 + 2$$

Parabool B:

Top (-2,3) en punt (0,1).

$$1 = c(0+2)^2 + 3$$

$$1 = 4c + 3$$

$$-2 = 4c$$

$$-\frac{1}{2} = c$$

$$y = -\frac{1}{2}(x+2)^2 + 3$$

Parabool C:

Top (-1,-4) en punt (0,-2).

$$-2 = c(0+1)^2 - 4$$

$$-2 = c - 4$$

$$2 = c$$

$$y = 2(x+1)^2 - 4$$

Parabool D:

Top (-1,0) en punt (0,1).

$$1 = c(0+1)^2 + 0$$

$$1 = c$$

$$y = (x+1)^2$$

b A en C:

$$-x^2 + 2 = 2(x+1)^2 - 4$$

$$-x^2 + 2 = 2x^2 + 4x + 2 - 4$$

$$3x^2 + 4x - 4 = 0$$

$$\left. \begin{array}{l} a = 3 \\ b = 4 \\ c = -4 \end{array} \right\} \begin{array}{l} D = 16 - 4 \cdot 3 \cdot -4 = 64 \\ \sqrt{D} = 8 \end{array}$$

$$x = \frac{-4+8}{6} = \frac{2}{3} \text{ of } x = \frac{-4-8}{6} = -2$$

$$\text{Als } x = \frac{2}{3}, \text{ dan } y = -\left(\frac{2}{3}\right)^2 + 2 = 1\frac{5}{9}.$$

$$\text{Als } x = -2, \text{ dan } y = -(-2)^2 + 2 = -2.$$

Snijpunten zijn $(\frac{2}{3}, 1\frac{5}{9})$ en $(-2, -2)$.

B en D:

$$-\frac{1}{2}(x+2)^2 + 3 = (x+1)^2$$

$$-\frac{1}{2}x^2 - 2x - 2 + 3 = x^2 + 2x + 1$$

$$3x^2 + 8x = 0$$

$$3x(x + \frac{8}{3}) = 0$$

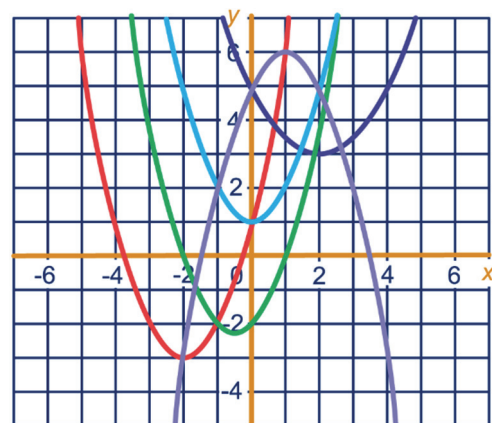
$$x = 0 \text{ of } x = -\frac{8}{3}$$

$$\text{Als } x = 0, \text{ dan } y = (0+1)^2 = 1.$$

$$\text{Als } x = -\frac{8}{3}, \text{ dan } y = \left(-\frac{8}{3} + 1\right)^2 = \frac{25}{9} = 2\frac{7}{9}.$$

Snijpunten zijn (0,1) en $(-\frac{8}{3}, 2\frac{7}{9})$.

2



3

$$y = \frac{1}{4}x^2 + 3x + 2$$

$$y = \frac{1}{4}(x^2 + 12x + 8)$$

$$y = \frac{1}{4}((x+6)^2 - 36 + 8)$$

$$y = \frac{1}{4}(x+6)^2 - 7$$

Top (-6,-7).

$$y = -2x^2 + 4x + 6$$

$$y = -2(x^2 - 2x - 3)$$

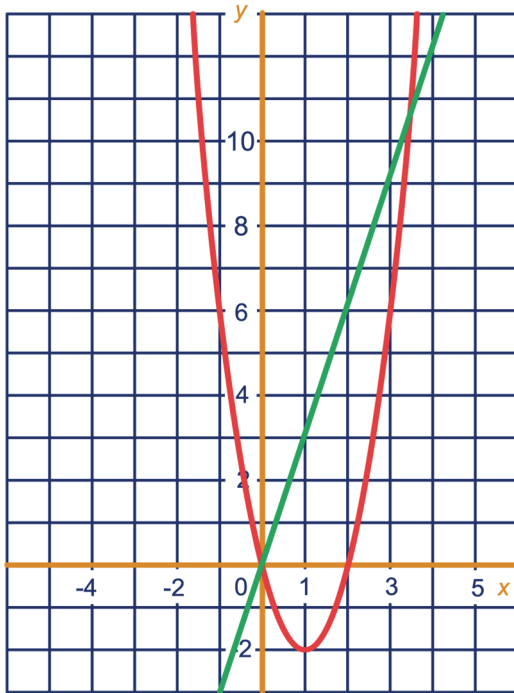
$$y = -2((x-1)^2 - 1 - 3)$$

$$y = -2(x-1)^2 + 8$$

Top(1,8).

4 a $0 = c(2-1)^2 - 2$
 $2 = c$
 $y = 2(x-1)^2 - 2$

bc



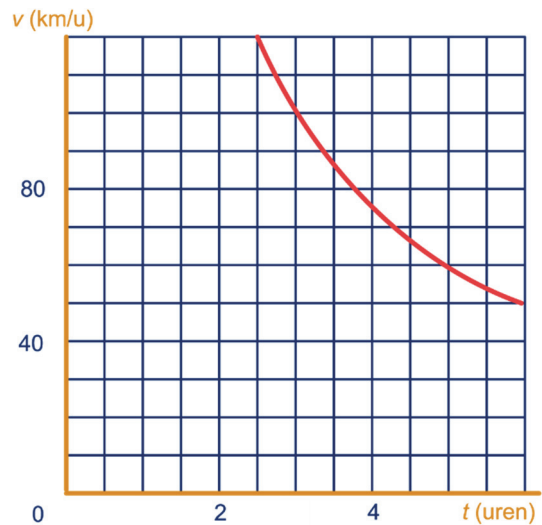
d $3x = 2(x-1)^2 - 2$
 $2x^2 - 7x = 0$
 $2x(x - 3\frac{1}{2}) = 0$
 $x = 0$ of $x = 3\frac{1}{2}$
 Als $x = 0$, dan $y = 3 \cdot 0 = 0$.
 Als $x = 3\frac{1}{2}$, dan $y = 3 \cdot 3\frac{1}{2} = 10\frac{1}{2}$.
 Snijpunten zijn $(0,0)$ en $(3\frac{1}{2}, 10\frac{1}{2})$.

e $3x + p = 2(x-1)^2 - 2$
 $2x^2 - 7x - p = 0$
 $a = 2$
 $b = -7$
 $c = -p$
 $D = 49 - 4 \cdot 2 \cdot -p = 49 + 8p$
 Eén oplossing als $D = 0$.
 $49 + 8p = 0$
 $8p = -49$
 $p = -\frac{49}{8} = -6\frac{1}{8}$

5 a $t = 300 : 120 = 2\frac{1}{2}$ uur

b $vt = 300$

c



d Als v klein is. De grafiek daalt steeds minder snel.

6 a $0 = a \cdot 100 - 5 \cdot 100^2$
 $50.000 = 100a$

$500 = a$

b Vanwege symmetrie wordt de grootste hoogte bereikt als $x = 50$.

Dan is $y = 500 \cdot 50 - 5 \cdot 50^2 = 12.500$, dus 12.500 meter.

7 $x^2 - 8x + 22 = 0$

$$\left. \begin{array}{l} a = 1 \\ b = -8 \\ c = 22 \end{array} \right\} D = 64 - 4 \cdot 1 \cdot 22 = -24$$

$D < 0$, dus geen oplossingen

$2x^2 = 5x - 3$

$2x^2 - 5x + 3 = 0$

$$\left. \begin{array}{l} a = 2 \\ b = -5 \\ c = 3 \end{array} \right\} D = 25 - 4 \cdot 2 \cdot 3 = 1$$

$$\sqrt{D} = 1$$

$x = \frac{5 \pm 1}{4} = 1\frac{1}{2}$ of $x = \frac{5-1}{4} = 1$

$3(x+2)^2 - 2x = 9$

$3x^2 + 10x + 3 = 0$

$$\left. \begin{array}{l} a = 3 \\ b = 10 \\ c = 3 \end{array} \right\} D = 100 - 4 \cdot 3 \cdot 3 = 64$$

$$\sqrt{D} = 8$$

$x = \frac{-10 \pm 8}{6} = -\frac{1}{3}$ of $x = \frac{-10-8}{6} = -3$

$$-5x^2 + 4x - \frac{4}{5} = 0$$

$$\left. \begin{array}{l} a = -5 \\ b = 4 \\ c = -\frac{4}{5} \end{array} \right\} D = 16 - 4 \cdot -5 \cdot -\frac{4}{5} = 0$$

$$x = -\frac{4}{-10} = \frac{2}{5}$$

8 a $y = -2x^2 + 12x$

$$y = -2(x^2 - 6x)$$

$$y = -2((x-3)^2 - 9)$$

$$y = -2(x-3)^2 + 18$$

Top (3,18).

b Voor $x = 3$, dan $y = 18$.

9 $x^2 - x\sqrt{2} - 4 = 0$

$$\left. \begin{array}{l} a = 1 \\ b = -\sqrt{2} \\ c = -4 \end{array} \right\} D = 2 - 4 \cdot 1 \cdot -4 = 18$$

$$\sqrt{D} = \sqrt{18} = 3\sqrt{2}$$

$$x = \frac{\sqrt{2} + 3\sqrt{2}}{2} = 2\sqrt{2} \text{ of } x = \frac{\sqrt{2} - 3\sqrt{2}}{2} = -\sqrt{2}$$

$$\sqrt{x^2 + 3x} = 3\sqrt{2}$$

$$x^2 + 3x = 18$$

$$x^2 + 3x - 18 = 0$$

$$(x-3)(x+6) = 0$$

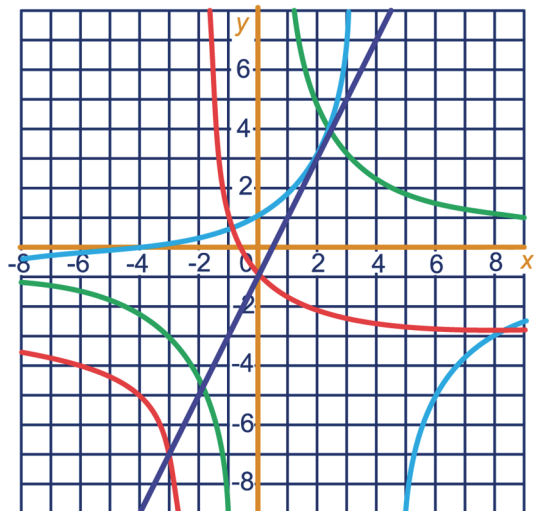
$$x = 3 \text{ of } x = -6$$

10 a horizontale asymptoot: $y = -3$
 verticale asymptoot: $x = -2$

horizontale asymptoot: $y = -1$
 verticale asymptoot: $x = 4$

horizontale asymptoot: $y = 0$
 verticale asymptoot: $x = 0$

bc



d $(x+2)(y+3) = 4$ en $y = 2x - 1$

$$(x+2)(2x-1+3) = 0$$

$$2x^2 + 6x = 0$$

$$2x(x+3) = 0$$

$$x = 0 \text{ of } x = -3$$

Als $x = 0$, dan $y = 0 - 1 = -1$.

Als $x = -3$, dan $y = -6 - 1 = -7$.

Snijpunten zijn (0,-1) en (-3,-7).

$$(x-4)(y+1) = -8 \text{ en } y = 2x - 1$$

$$(x-4)(2x-1+1) = -8$$

$$2x^2 - 8x + 8 = 0$$

$$x^2 - 4x + 4 = 0$$

$$(x-2)^2 = 0$$

$$x = 2$$

Als $x = 2$, dan $y = 4 - 1 = 3$.

Snijpunt is (2,3).

$$xy = 9 \text{ en } y = 2x - 1$$

$$x(2x-1) = 9$$

$$2x^2 - x - 9 = 0$$

$$\left. \begin{array}{l} a = 2 \\ b = -1 \\ c = -9 \end{array} \right\} D = 1 - 4 \cdot 2 \cdot -9 = 73$$

$$\sqrt{D} = \sqrt{73}$$

$$x = \frac{1 + \sqrt{73}}{4} = \frac{1}{4} + \frac{1}{4}\sqrt{73} \text{ of } x = \frac{1}{4} - \frac{1}{4}\sqrt{73}$$

Als $x = \frac{1}{4} + \frac{1}{4}\sqrt{73}$, dan $y = -\frac{1}{2} + \frac{1}{2}\sqrt{73}$.

Als $x = \frac{1}{4} - \frac{1}{4}\sqrt{73}$, dan $y = -\frac{1}{2} - \frac{1}{2}\sqrt{73}$.

Snijpunten zijn $(\frac{1}{4} + \frac{1}{4}\sqrt{73}, -\frac{1}{2} + \frac{1}{2}\sqrt{73})$ en $(\frac{1}{4} - \frac{1}{4}\sqrt{73}, -\frac{1}{2} - \frac{1}{2}\sqrt{73})$.

11 horizontale asymptoot: $y = 0$
 verticale asymptoot: $x = 0$
 Punt op hyperbool: (3,-3)
 $xy = 3 \cdot -3 = -9$

horizontale asymptoot: $y = 1$

verticale asymptoot: $x = -1$

Punt op hyperbool: (2,3)

$$(x+1)(y-1) = c$$

$$(2+1)(3-1) = c$$

$$6 = c$$

$$(x+1)(y-1) = 6$$

horizontale asymptoot: $y = -3$

verticale asymptoot: $x = 0$

Punt op hyperbool: (2,1)

$$x(y+3) = c$$

$$2 \cdot (1+3) = c$$

$$8 = c$$

$$x(y+3) = 8$$

12 $x^2 + 3x + p = 0$

$$\left. \begin{array}{l} a = 1 \\ b = 3 \\ c = p \end{array} \right\} D = 9 - 4 \cdot 1 \cdot p = 9 - 4p$$

Twee oplossingen als $D > 0$:

$$9 - 4p > 0$$

$$9 > 4p$$

$$2\frac{1}{4} > p$$

Dus twee oplossingen als $p < 2\frac{1}{4}$.

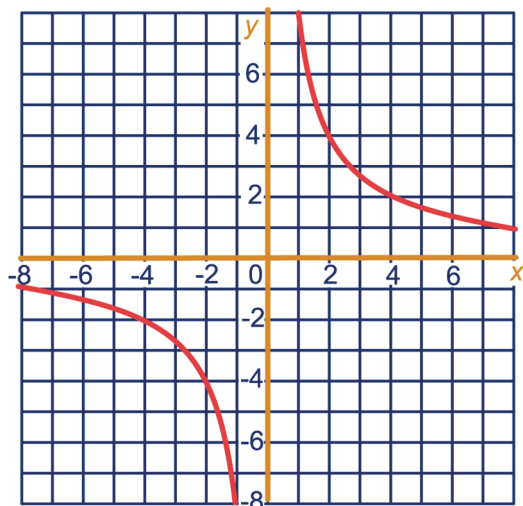
Eén oplossing als $D = 0$:

$$p = 2\frac{1}{4}$$

Geen oplossingen als $D < 0$:

$$p > 2\frac{1}{4}$$

13 a



b $xy = 8$ en $y = -x + k$

$$x(-x + k) = 8$$

$$-x^2 + kx - 8 = 0$$

$$\left. \begin{array}{l} a = -1 \\ b = k \\ c = -8 \end{array} \right\} D = k^2 - 4 \cdot (-1) \cdot (-8) = k^2 - 32$$

Raken $\Rightarrow D = 0$, dus

$$k^2 - 32 = 0$$

$$k^2 = 32$$

$$k = \sqrt{32} = 4\sqrt{2} \text{ of } k = -4\sqrt{2}$$

Vergelijking raaklijnen:

$$y = -x + 4\sqrt{2} \text{ en } y = -x - 4\sqrt{2}$$

14 $2(x+3)^2 - 4 = -3x + 1$

$$2x^2 + 15x + 13 = 0$$

$$\left. \begin{array}{l} a = 2 \\ b = 15 \\ c = 13 \end{array} \right\} D = 225 - 4 \cdot 2 \cdot 13 = 121$$

$$\sqrt{D} = 11$$

$$x = \frac{-15+11}{4} = -1 \text{ of } x = \frac{-15-11}{4} = -6\frac{1}{2}$$

Als $x = -1$, dan $y = 3 + 1 = 4$.

Als $x = -6\frac{1}{2}$, dan $y = 19\frac{1}{2} + 1 = 20\frac{1}{2}$.

Snijpunten zijn $(-1, 4)$ en $(-6\frac{1}{2}, 20\frac{1}{2})$.

15 oppervlakte driehoeken = $2 \cdot x(8-x)$
oppervlakte driehoeken = $\frac{1}{4} \cdot 8 \cdot 8 = 16$

Vergelijking:

$$2 \cdot x(8-x) = 16$$

$$2x^2 - 16x + 16 = 0$$

$$x^2 - 8x + 8 = 0$$

$$\left. \begin{array}{l} a = 1 \\ b = -8 \\ c = 8 \end{array} \right\} D = 64 - 4 \cdot 1 \cdot 8 = 32$$

$$\sqrt{D} = \sqrt{32} = 4\sqrt{2}$$

$$x = \frac{8+4\sqrt{2}}{2} = 4 + 2\sqrt{2} \text{ cm of } x = 4 - 2\sqrt{2} \text{ cm}$$

Allebei de oplossingen voldoen, want $0 < x < 8$.

16 Oppervlakte balk is
 $2(x(x+4) + x(x+3) + (x+4)(x+3)) =$
 $2(3x^2 + 14x + 12) = 6x^2 + 28x + 24$.

Vergelijking:

$$6x^2 + 28x + 24 = 162$$

$$6x^2 + 28x - 138 = 0$$

$$\left. \begin{array}{l} a = 6 \\ b = 28 \\ c = -138 \end{array} \right\} D = 28^2 - 4 \cdot 6 \cdot (-138) = 4096$$

$$\sqrt{D} = \sqrt{4096} = 64$$

$$x = \frac{-28+64}{12} = 3 \text{ of } x = \frac{-28-64}{12} = -7\frac{2}{3}$$

Alleen $x = 3$ voldoet, omdat $x > 0$ moet zijn.

17 a $50t - 5t^2 = 0$

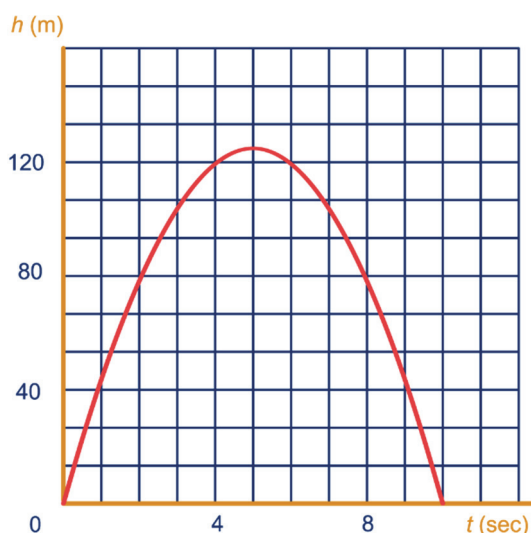
$$5t(10-t) = 0$$

$$t = 0 \text{ of } t = 10$$

Dus de vlucht duurt $10 - 0 = 10$ sec.

b Maximale hoogte wordt bereikt na 5 sec.

$$h = 50 \cdot 5 - 5 \cdot 5^2 = 250 - 125 = 125 \text{ m}$$

c

d $50t - 5t^2 > 113,75$

$$0 > 5t^2 - 50t + 113,75$$

$$t^2 - 10t + 22,75 < 0$$

$$(t - 3,5)(t - 6,5) < 0$$

$$3,5 < t < 6,5$$

Dus tussen de 3,5 en 6,5 seconde is de hoogte van de vuurpijl meer dan 113,75 m.

18 a $(10 + 2)^2 - 10 - 10 = 124$ stippen

b $(n + 2)^2 - 2n = n^2 + 2n + 4$

c $n^2 + 2n + 4 = 10.204$

$$n^2 + 2n - 10.200 = 0$$

$$(n - 100)(n + 102) = 0$$

$$n = 100 \text{ of } n = -102$$

Alleen $n = 100$ voldoet, omdat $n > 0$.

19 a $\frac{18}{45} = \frac{x}{y}$

$$18y = 45x$$

$$y = 2\frac{1}{2}x$$

Breedte van de rechthoek is

$$45 - y = 45 - 2\frac{1}{2}x.$$

$$O = x \cdot (45 - 2\frac{1}{2}x) = 45x - 2\frac{1}{2}x^2$$

b $O = 45x - 2\frac{1}{2}x^2$

$$O = -2\frac{1}{2}x^2 + 45x$$

$$O = -2\frac{1}{2}(x^2 - 18x)$$

$$O = -2\frac{1}{2}((x - 9)^2 - 81)$$

$$O = -2\frac{1}{2}((x - 9)^2) + 202\frac{1}{2}$$

De oppervlakte is maximaal als $x = 9$.

c De oppervlakte is dan $202\frac{1}{2}$.